KINEMATICS OF THE PLANAR QUADRILATERAL MECHANISM

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Abstract: This paper presents an original method to determine the kinematic parameters at the linked quadrilateral mechanism. It is starting with a trigonometric method, which has the advantage to determine very quickly the position angles. The velocities can be determined faster using a geometric method. This method is developed and for the accelerations determinations. The (proposed) geometric method, determines first the kinematic parameters of the internal couple (B) and then the rotation angles with their derivatives. Secondary, the paper presents the determination of the efficiency of this mechanism. Determines and dynamic coefficient, D. With this one proposes two yields; the mechanical efficiency and the dynamic efficiency.

Keywords: 3R dyad, kinematic parameters, efficiency, dynamic coefficient

INTRODUCTION

The paper presents an original method to determine the kinematic parameters at the linked quadrilateral mechanism [3-4].

It is starting with a trigonometric method, which has the advantage to determine very quickly the position angles.

The velocities can be determined faster using a geometric method. This method is developed and for the accelerations determinations. The (proposed) geometric method, determines first the kinematic parameters of the internal couple (B) and then the rotation angles with their derivatives.

Secondary, the paper presents the determination of the efficiency of this mechanism. Determines and dynamic coefficient, D.

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THE KINEMATICS OF THE PLANAR QUADRILATERAL MECHANISM

The kinematic schema of a planar quadrilateral mechanism [1-2] can be seen in the Figure 1.

The following kinematic parameters considered known:

$x_O; y_O; x_C; y_C; l_1; l_2; l_3; \varphi_1; \omega_1 = ct.$; where the positions parameters $(x_O; y_O)$ of the couple O are zero (if we set the rectangular system in O); l_1 is the length of the crank 1; l_2 is the length of the connecting rod 2; l_3 is the length of the rocker 3; φ_1 is the crank angle position, and ω_1 is its angular velocity. The C couple coordinates $(x_C; y_C)$ are also known.

We must determine now the cinematic parameters which give the positions of the rod (φ_2) and the crank (φ_3).

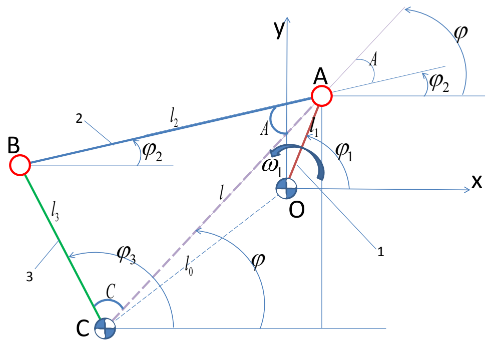

Fig. 1. Kinematic schema of a planar quadrilateral mechanism

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$$
\begin{cases}
\begin{cases} x_A = l_1 \cdot \cos \varphi_1 \\ y_A = l_1 \cdot \sin \varphi_1 \end{cases}
\begin{cases} \dot{x}_A = -l_1 \cdot \sin \varphi_1 \cdot \omega_1 \\ \dot{y}_A = l_1 \cdot \cos \varphi_1 \cdot \omega_1 \end{cases} \\
\begin{cases} \ddot{x}_A = -l_1 \cdot \cos \varphi_1 \cdot \omega_1^2 \\ \ddot{y}_A = -l_1 \cdot \sin \varphi_1 \cdot \omega_1^2 \end{cases} \\
l^2 = (x_A - x_C)^2 + (y_A - y_C)^2 \Rightarrow \\
l = \sqrt{l^2} = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2} \\
\cos A = \dfrac{l^2 + l_2^2 - l_3^2}{2 \cdot l \cdot l_2} \Rightarrow A = \arccos(\cos A); \\
\cos C = \dfrac{l^2 + l_3^2 - l_2^2}{2 \cdot l \cdot l_3} \Rightarrow C = \arccos(\cos C) \\
\begin{cases} \cos \varphi = \dfrac{x_A - x_C}{l} \\ \sin \varphi = \dfrac{y_A - y_C}{l} \end{cases} \Rightarrow \\
\Rightarrow \varphi = sign(\sin \varphi) \cdot \arccos(\cos \varphi); \\
\begin{cases} \varphi_2 = \varphi - A \\ \varphi_3 = \varphi + C \end{cases}
\begin{cases} x_B = x_C + l_3 \cdot \cos \varphi_3 \\ y_B = y_C + l_3 \cdot \sin \varphi_3 \end{cases}
\end{cases}
\qquad (1)
$$

DETERMINING THE POSITIONS

It is starting with a trigonometric method, which has the advantage to determine very quickly the position angles (the system 1). First it determines the cinematic parameters of the A couple ($x_A; y_A$), and their first two derivatives ($\dot{x}_A; \dot{y}_A; \ddot{x}_A; \ddot{y}_A$). Second, one finds the variable length (l) between C and A, and it determines the position angle (φ) of the CA vector. We also determine the angles A and C from the triangle ABC. Finally we found the positions of the rod (φ_2) and the crank (φ_3).

$$
\begin{cases}
\begin{cases} (x_B - x_C)^2 + (y_B - y_C)^2 = l_3^2 \\ (x_B - x_A)^2 + (y_B - y_A)^2 = l_2^2 \end{cases} \Rightarrow \\
\Rightarrow \begin{cases} (x_B - x_C) \cdot \dot{x}_B + (y_B - y_C) \cdot \dot{y}_B = 0 \\ (x_B - x_A) \cdot \dot{x}_B + (y_B - y_A) \cdot \dot{y}_B = \\ = (x_B - x_A) \cdot \dot{x}_A + (y_B - y_A) \cdot \dot{y}_A \end{cases} \\
a_{11} = x_B - x_C; \quad a_{12} = y_B - y_C; \\
a_{21} = x_B - x_A; \\
a_{22} = y_B - y_A; \quad b_1 = 0; \\
b_2 = a_{21} \cdot \dot{x}_A + a_{22} \cdot \dot{y}_A \\
\begin{cases} a_{11} \cdot \dot{x}_B + a_{12} \cdot \dot{y}_B = b_1 \\ a_{21} \cdot \dot{x}_B + a_{22} \cdot \dot{y}_B = b_2 \end{cases} \Rightarrow \\
\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}; \\
\Delta_{\dot{x}_B} = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = b_1 \cdot a_{22} - a_{12} \cdot b_2 \\
\Delta_{\dot{y}_B} = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = a_{11} \cdot b_2 - a_{21} \cdot b_1; \\
\dot{x}_B = \dfrac{\Delta_{\dot{x}_B}}{\Delta}; \quad \dot{y}_B = \dfrac{\Delta_{\dot{y}_B}}{\Delta}
\end{cases}
\qquad (2)
$$

DETERMINING THE VELOCITIES OF THE COUPLE B

The velocities ($\dot{x}_B; \dot{y}_B$) can be determined faster using a geometric method (the system 2). First, we write the equations of the circles which have the radius l_3 and l_2. They intersect in B. This system does not have to be solve because the solutions are already known. It will be derivative two times to obtain the systems of velocity and accelerations. These systems are linear and are solved by determinants. At 2.2 we obtain the velocities ($\dot{x}_B; \dot{y}_B$), and at 2.3 determine the accelerations ($\ddot{x}_B; \ddot{y}_B$).

DETERMINING THE ACCELERATIONS OF THE COUPLE B

The accelerations $(\ddot{x}_B;\ddot{y}_B)$ can be determined faster using a geometric method (the system 3).

$$\begin{cases} \begin{cases} (x_B - x_C)\cdot\ddot{x}_B + (y_B - y_C)\cdot\ddot{y}_B = \\ = -\dot{x}_B^2 - \dot{y}_B^2 \\ (x_B - x_A)\cdot\ddot{x}_B + (y_B - y_A)\cdot\ddot{y}_B = \\ = a_{21}\cdot\ddot{x}_A + a_{22}\cdot\ddot{y}_A - \dot{a}_{21}^2 - \dot{a}_{22}^2 \end{cases} \\ c_1 = -\dot{x}_B^2 - \dot{y}_B^2; \\ c_2 = a_{21}\cdot\ddot{x}_A + a_{22}\cdot\ddot{y}_A - \dot{a}_{21}^2 - \dot{a}_{22}^2 \\ \begin{cases} a_{11}\cdot\ddot{x}_B + a_{12}\cdot\ddot{y}_B = c_1 \\ a_{21}\cdot\ddot{x}_B + a_{22}\cdot\ddot{y}_B = c_2 \end{cases} \Rightarrow \\ \Delta_{\ddot{x}_B} = \begin{vmatrix} c_1 & a_{12} \\ c_2 & a_{22} \end{vmatrix} = c_1\cdot a_{22} - a_{12}\cdot c_2 \\ \Delta_{\ddot{y}_B} = \begin{vmatrix} a_{11} & c_1 \\ a_{21} & c_2 \end{vmatrix} = a_{11}\cdot c_2 - a_{21}\cdot c_1; \\ \ddot{x}_B = \dfrac{\Delta_{\ddot{x}_B}}{\Delta};\quad \ddot{y}_B = \dfrac{\Delta_{\ddot{y}_B}}{\Delta} \end{cases} \tag{3}$$

DETERMINING THE ANGULAR VELOCITIES AND ACCELERATIONS

The angular velocities $(\omega_2;\omega_3)$ and accelerations $(\varepsilon_2;\varepsilon_3)$ can be determined now, faster, using the vectorial method (system 4).

$$\begin{cases} \begin{cases} x_A - x_B = l_2\cdot\cos\varphi_2 \\ y_A - y_B = l_2\cdot\sin\varphi_2 \end{cases} \\ \begin{cases} \dot{x}_A - \dot{x}_B = -l_2\cdot\sin\varphi_2\cdot\omega_2 \mid\cdot(-\sin\varphi_2) \\ \dot{y}_A - \dot{y}_B = l_2\cdot\cos\varphi_2\cdot\omega_2 \mid\cdot(\cos\varphi_2) \end{cases} \Rightarrow \\ \Rightarrow \omega_2 = \dfrac{(\dot{y}_A - \dot{y}_B)\cdot\cos\varphi_2 - (\dot{x}_A - \dot{x}_B)\cdot\sin\varphi_2}{l_2} \\ \begin{cases} \ddot{x}_A - \ddot{x}_B = -l_2\cdot\sin\varphi_2\cdot\varepsilon_2 - l_2\cdot\cos\varphi_2\cdot\omega_2^2 \mid\cdot(-\sin\varphi_2) \\ \ddot{y}_A - \ddot{y}_B = l_2\cdot\cos\varphi_2\cdot\varepsilon_2 - l_2\cdot\sin\varphi_2\cdot\omega_2^2 \mid\cdot(\cos\varphi_2) \end{cases} \Rightarrow \\ \Rightarrow \varepsilon_2 = \dfrac{(\ddot{y}_A - \ddot{y}_B)\cdot\cos\varphi_2 - (\ddot{x}_A - \ddot{x}_B)\cdot\sin\varphi_2}{l_2} \\ \begin{cases} x_B - x_C = l_3\cdot\cos\varphi_3 \\ y_B - y_C = l_3\cdot\sin\varphi_3 \end{cases} \\ \begin{cases} \dot{x}_B - \dot{x}_C = -l_3\cdot\sin\varphi_3\cdot\omega_3 \mid\cdot(-\sin\varphi_3) \\ \dot{y}_B - \dot{y}_C = l_3\cdot\cos\varphi_3\cdot\omega_3 \mid\cdot(\cos\varphi_3) \end{cases} \Rightarrow \\ \Rightarrow \omega_3 = \dfrac{(\dot{y}_B - \dot{y}_C)\cdot\cos\varphi_3 - (\dot{x}_B - \dot{x}_C)\cdot\sin\varphi_3}{l_3} \\ \begin{cases} \ddot{x}_B - \ddot{x}_C = -l_3\cdot\sin\varphi_3\cdot\varepsilon_3 - l_3\cdot\cos\varphi_3\cdot\omega_3^2 \mid\cdot(-\sin\varphi_3) \\ \ddot{y}_B - \ddot{y}_C = l_3\cdot\cos\varphi_3\cdot\varepsilon_3 - l_3\cdot\sin\varphi_3\cdot\omega_3^2 \mid\cdot(\cos\varphi_3) \end{cases} \Rightarrow \\ \Rightarrow \varepsilon_3 = \dfrac{(\ddot{y}_B - \ddot{y}_C)\cdot\cos\varphi_3 - (\ddot{x}_B - \ddot{x}_C)\cdot\sin\varphi_3}{l_3} \end{cases} \tag{4}$$

THE EFFICIENCY OF THE PLANAR QUADRILATERAL MECHANISM

The efficiency of a planar quadrilateral mechanism can be determined starting from the forces and velocities repartition, (Figure 2).

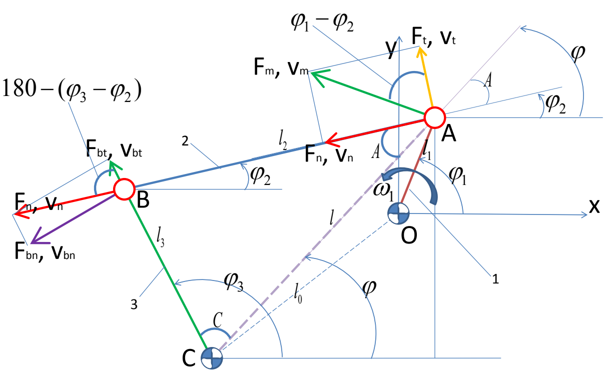

Fig. 2. *Forces and velocities repartition of a planar quadrilateral mechanism*

The system (5) presents the relationships which give the forces and the velocities on the planar quadrilateral mechanism. The driving force F_m is perpendicular on the crank 1 in A.

Its component along the connecting rod

(the bar 2) F_n, gives the normal component F_{bn}. F_{bn} is perpendicular on the rocker 3 in B.

These forces give the dynamic velocities which are similar with the forces.

The forces are always the same, but the velocities (the dynamic velocities) are different than the kinematics velocities.

For this reason the dynamic efficiency will be different than the mechanical yield.

$$\begin{cases} \begin{cases} F_n = F_m \cdot \sin(\varphi_1 - \varphi_2) \\ v_n = v_m \cdot \sin(\varphi_1 - \varphi_2) \end{cases} \\ \begin{cases} F_B \equiv F_b = F_n \cdot \sin[\pi - (\varphi_3 - \varphi_2)] = \\ = F_m \cdot \sin(\varphi_1 - \varphi_2) \cdot \sin(\varphi_3 - \varphi_2) \\ v_B^D \equiv v_b = v_n \cdot \sin[\pi - (\varphi_3 - \varphi_2)] = \\ = v_m \cdot \sin(\varphi_1 - \varphi_2) \cdot \sin(\varphi_3 - \varphi_2) \end{cases} \\ \omega_3 = \frac{l_1 \cdot \sin(\varphi_1 - \varphi_2) \cdot \omega_1}{l_3 \cdot \sin(\varphi_3 - \varphi_2)} \Rightarrow \\ v_B = l_3 \cdot \omega_3 = \frac{l_1 \cdot \omega_1 \cdot \sin(\varphi_1 - \varphi_2)}{\sin(\varphi_3 - \varphi_2)} = \\ = \frac{v_m \cdot \sin(\varphi_1 - \varphi_2)}{\sin(\varphi_3 - \varphi_2)} \\ v_B^D = D \cdot v_B \Leftrightarrow \\ v_m \cdot \sin(\varphi_1 - \varphi_2) \cdot \sin(\varphi_3 - \varphi_2) = \\ = D \cdot \frac{v_m \cdot \sin(\varphi_1 - \varphi_2)}{\sin(\varphi_3 - \varphi_2)} \Rightarrow \\ D = \sin^2(\varphi_3 - \varphi_2) \\ \eta_i = \frac{P_3}{P_1} = \frac{F_B \cdot v_B}{F_m \cdot v_m} = \\ = \frac{F_m \cdot \sin(\varphi_1 - \varphi_2) \cdot \sin(\varphi_3 - \varphi_2) \cdot \frac{v_m \cdot \sin(\varphi_1 - \varphi_2)}{\sin(\varphi_3 - \varphi_2)}}{F_m \cdot v_m} = \\ = \sin^2(\varphi_1 - \varphi_2) \\ \eta_i^D = \frac{P_3^D}{P_1} = \frac{F_B \cdot v_B^D}{F_m \cdot v_m} = \\ = \frac{F_m \cdot \sin(\varphi_1 - \varphi_2) \cdot \sin(\varphi_3 - \varphi_2) \cdot v_m \cdot \sin(\varphi_1 - \varphi_2) \cdot \sin(\varphi_3 - \varphi_2)}{F_m \cdot v_m} = \\ = \sin^2(\varphi_3 - \varphi_2) \cdot \sin^2(\varphi_1 - \varphi_2) = D \cdot \eta_i \end{cases} \quad (5)$$

CONCLUSIONS

The presented method is the most elegant and direct method to determine the kinematics planar quadrilateral mechanism.

Relationships used by this method allow and the determination of the dynamic system vibration. In the dynamic kinematics the constant rotation speed $\omega_1 = c$. gets a variable value $\omega_1^D = D \cdot \omega_1$. We consider that the rotation speed (ω_1) of the crank is a constant for a constant rotation speed (n) (relation 6).

$$\omega_1 = 2 \cdot \pi \cdot \nu = 2 \cdot \pi \cdot \frac{n}{60} = \frac{\pi \cdot n}{30} \quad (6)$$

In fact ω_1 varies with the position of the crank angle (φ) (system 7, [3-4]).

$$\begin{cases} \omega_{1m} = D \cdot \frac{\pi \cdot n}{30} = \sin^2(\varphi_3 - \varphi_2) \cdot \frac{\pi \cdot n}{30}; \\ J_m^* = J_{medium}^* = \frac{J_{\min}^* + J_{\max}^*}{2}; \\ \omega_1 = \omega_{1m} \cdot \sqrt{\frac{J_m^*}{J^*}} = \\ = \sin^2(\varphi_3 - \varphi_2) \cdot \frac{\pi \cdot n}{30} \cdot \sqrt{\frac{J_m^*}{J^*}}; \\ \varepsilon_1 = -\frac{\omega_1^2}{2} \cdot \frac{J^{*'}}{J^*} = -\frac{\omega_1^2}{2} \cdot \frac{\frac{dJ^*}{d\varphi}}{J^*} \end{cases} \quad (7)$$

J^* is the mechanical moment of inertia reduced to crank; $J_m^* = J_{medium}^*$.

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