KINETOSTATIC OF THE 3R DYAD (OR 2R MODULE)

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Abstract: This paper presents a method to determine the kinetostatic parameters at the 3R dyad. One proposes to determine the forces from joints: R_B, R_D, R_{23}. To generalize the method and to the 2R robots, are introduced and the two moments M_1, M_2. This (2R module) is the principal from the anthropomorphous rotation robotic structures and mechatronic structures. Figure 1 shows a schematic diagram of the 3R dyad minimum kinetostatic (determination of static forces); (loaded with the inertia forces, considered external forces). For if there are additional external forces, such as technological resistances will be added as well.

Keywords: 3R dyad, kinetostatic parameters, external forces, internal forces

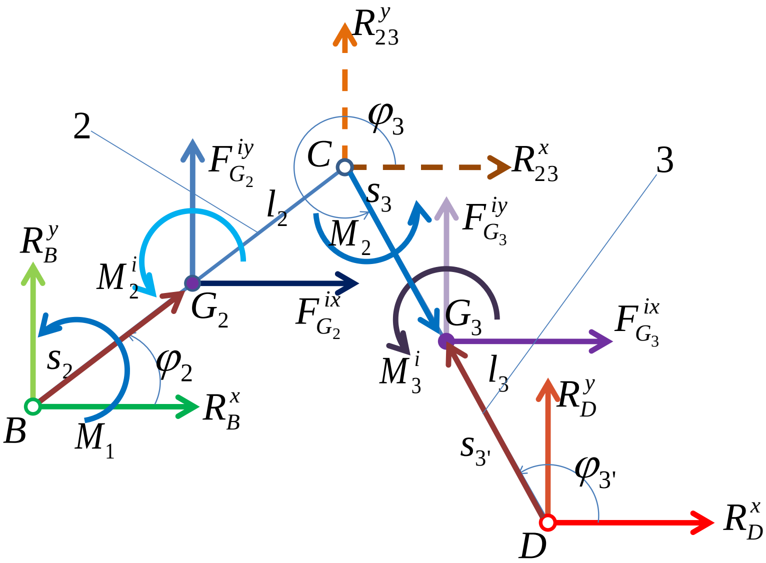

Fig. 1. *The kinetostatic parameters to a 3R dyad*

1. INTRODUCTION

In this paper it presents a method able to determine the kinetostatic parameters to a 3R dyad (see the Figure 1) [1-4].

To generalize the method and to the 2R robots, are introduced and the two moments M_1, M_2. This 2R module, is the principal from the android rotation robotic structures and mechatronic structures [1], [3].

The 3R dyad has two elements, noted with 2 and 3. Their lengths are l_2 and l_3.

If the 3R dyad is coupling to a 4R mechanism, we note the forces which give the entry into dyad, with R_{12} and R_{03}. In case the structure 2-3 is using to a robot or to another mechanism, we note the entrance forces, with R_B and R_D.

One proposes to determine the forces from joints: R_B, R_D, R_{23}.

Figure 1 shows a schematic diagram of the 3R dyad minimum kinetostatic (loaded with the inertia forces, considered external forces).

For if there are additional external forces, such as technological resistances will be added as well.

One can consider and the forces of gravity, if mechanism operates strictly vertically and working speeds are low [1-4].

2. DETERMINING THE FORCES FROM JOINTS

The joints forces represent the interior loads (internal forces).

One proposes to determine these (internal) forces.

We start with the internal force R_B, which is divided in two components in a cartesian planar system: R_B^x, R_B^y.

If external forces are known in general (are given, determined, calculated), internal forces (reactions of kinematic couplings) results from the balance of forces and moments of the dyad [2], [3], [4].

To start [3] we are writing an equation representing the sum of the moments from element 2 in relation to the point C, and another relationship which represent the sum of all moments from entire dyad, in relation to the point D (system 1).

$$\begin{cases} \sum M_C^{(2)} = 0 \Rightarrow R_B^x \cdot (y_C - y_B) - \\ R_B^y \cdot (x_C - x_B) + M_1 + \\ + F_{G_2}^{ix} \cdot (y_C - y_{G_2}) - \\ - F_{G_2}^{iy} \cdot (x_C - x_{G_2}) + M_2^i = 0 \\ \sum M_D^{(2,3)} = 0 \Rightarrow R_B^x \cdot (y_D - y_B) - \\ - R_B^y \cdot (x_D - x_B) + M_1 + \\ + F_{G_2}^{ix} \cdot (y_D - y_{G_2}) - M_2^i + M_2 + \\ - F_{G_2}^{iy} \cdot (x_D - x_{G_2}) + M_3^i + \\ + F_{G_3}^{ix} \cdot (y_D - y_{G_3}) - \\ - F_{G_3}^{iy} \cdot (x_D - x_{G_3}) = 0 \end{cases} \quad (1)$$

The two equations are rewritten in the form of the system (2).

$$\begin{cases} (y_C - y_B) \cdot R_B^x - (x_C - x_B) \cdot R_B^y = -M_1 - \\ - F_{G_2}^{ix} \cdot (y_C - y_{G_2}) + F_{G_2}^{iy} \cdot (x_C - x_{G_2}) - M_2^i \\ (y_D - y_B) \cdot R_B^x - (x_D - x_B) \cdot R_B^y = \\ = -M_1 - F_{G_2}^{ix} \cdot (y_D - y_{G_2}) + \\ + F_{G_2}^{iy} \cdot (x_D - x_{G_2}) - M_2^i - M_2 - \\ - F_{G_3}^{ix} \cdot (y_D - y_{G_3}) + F_{G_3}^{iy} \cdot (x_D - x_{G_3}) - M_3^i \end{cases} \quad (2)$$

System (2) can be arranged as a linear system (3) by two equations with two unknowns $R_{12}^x \equiv R_B^x;\ \ R_{12}^y \equiv R_B^y$, with the coefficients, given from system (4).

$$\begin{cases} \begin{cases} a_{11} \cdot R_{12}^x + a_{12} \cdot R_{12}^y = a_1 \\ a_{21} \cdot R_{12}^x + a_{22} \cdot R_{12}^y = a_2 \end{cases} \\ or \\ \begin{cases} a_{11} \cdot R_B^x + a_{12} \cdot R_B^y = a_1 \\ a_{21} \cdot R_B^x + a_{22} \cdot R_B^y = a_2 \end{cases} \end{cases} \quad (3)$$

$$\begin{cases} a_{11} = y_C - y_B;\ \ a_{12} = -(x_C - x_B); \\ a_1 = -M_1 - F_{G_2}^{ix} \cdot (y_C - y_{G_2}) + \\ + F_{G_2}^{iy} \cdot (x_C - x_{G_2}) - M_2^i \\ a_{21} = y_D - y_B;\ \ a_{22} = -(x_D - x_B); \\ a_2 = -M_1 - F_{G_2}^{ix} \cdot (y_D - y_{G_2}) + F_{G_2}^{iy} \cdot (x_D - x_{G_2}) - \\ - M_2^i - M_2 - - F_{G_3}^{ix} \cdot (y_D - y_{G_3}) + \\ + F_{G_3}^{iy} \cdot (x_D - x_{G_3}) - M_3^i \end{cases} \quad (4)$$

Solutions of the system (3) will be given by system (5).

$$\begin{cases}\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\cdot a_{22} - a_{12}\cdot a_{21} \\ \Delta_x = \begin{vmatrix} a_1 & a_{12} \\ a_2 & a_{22} \end{vmatrix} = a_{22}\cdot a_1 - a_{12}\cdot a_2 \\ \Delta_y = \begin{vmatrix} a_{11} & a_1 \\ a_{21} & a_2 \end{vmatrix} = a_{11}\cdot a_2 - a_{21}\cdot a_1 \\ R_B^x \equiv R_{12}^x = \dfrac{\Delta_x}{\Delta} = \dfrac{a_{22}\cdot a_1 - a_{12}\cdot a_2}{a_{11}\cdot a_{22} - a_{12}\cdot a_{21}}; \\ R_B^y \equiv R_{12}^y = \dfrac{\Delta_y}{\Delta} = \dfrac{a_{11}\cdot a_2 - a_{21}\cdot a_1}{a_{11}\cdot a_{22} - a_{12}\cdot a_{21}} \end{cases} \quad (5)$$

Further determine other two internal forces, R_{03}^x şi R_{03}^y, or (R_D^x şi R_D^y).

Next we write the sum of all forces on the dyad (2,3) designed separately, first on the x axis and then on the y axis, (see the system 6).

$$\begin{cases}\sum F_x^{(2,3)} = 0 \Rightarrow \\ \Rightarrow R_{12}^x + F_{G_2}^{ix} + F_{G_3}^{ix} + R_{03}^x = 0 \Rightarrow \\ \Rightarrow R_D^x \equiv R_{03}^x = -R_{12}^x - F_{G_2}^{ix} - F_{G_3}^{ix} \\ \sum F_y^{(2,3)} = 0 \Rightarrow \\ \Rightarrow R_{12}^y + F_{G_2}^{iy} + F_{G_3}^{iy} + R_{03}^y = 0 \Rightarrow \\ \Rightarrow R_D^y \equiv R_{03}^y = -R_{12}^y - F_{G_2}^{iy} - F_{G_3}^{iy} \end{cases} \quad (6)$$

$$\begin{cases}\begin{cases}\sum F_x^{(2)} = 0 \Rightarrow R_{12}^x + F_{G_2}^{ix} - \\ - R_{23}^x = 0 \Rightarrow R_{23}^x = R_{12}^x + F_{G_2}^{ix} \\ \sum F_y^{(2)} = 0 \Rightarrow R_{12}^y + F_{G_2}^{iy} - \\ - R_{23}^y = 0 \Rightarrow R_{23}^y = R_{12}^y + F_{G_2}^{iy} \end{cases} \\ or \begin{cases}\sum F_x^{(3)} = 0 \Rightarrow R_{23}^x + F_{G_3}^{ix} + \\ + R_D^x = 0 \Rightarrow R_{23}^x = -F_{G_3}^{ix} - R_D^x \\ \sum F_y^{(3)} = 0 \Rightarrow R_{23}^y + F_{G_3}^{iy} + \\ + R_D^y = 0 \Rightarrow R_{23}^y = -F_{G_3}^{iy} - R_D^y \end{cases}\end{cases} \quad (7)$$

For the last two scalar components of the internal force from the joint C, one writes a new balance of forces on element 2 (for example), designed separately on axes x and y (system 7).

We obtained directly the internal forces R_{23}^x and R_{23}^y. Their opposites, R_{32}^x and R_{32}^y, they will be equal but opposite directed their, or in other words will have the same value but opposite sign [3].

For that all kinetostatic calculations of the 3R dyad to be possible, must be determined in advance, the forces and moments of inertia, separately for each element of the dyad. These are called „the group of the inertial forces”, and are expressed with the relations system (8).

$$\begin{cases}\begin{cases}F_{G_2}^{ix} = -m_2\cdot\ddot{x}_{G_2} \\ F_{G_2}^{iy} = -m_2\cdot\ddot{y}_{G_2} \\ M_2^i = -J_{G_2}\cdot\varepsilon_2\end{cases} \quad \begin{cases}F_{G_3}^{ix} = -m_3\cdot\ddot{x}_{G_3} \\ F_{G_3}^{iy} = -m_3\cdot\ddot{y}_{G_3} \\ M_3^i = -J_{G_3}\cdot\varepsilon_3\end{cases} \\ \begin{cases}x_{G_2} = x_B + s_2\cdot\cos\varphi_2 \\ y_{G_2} = y_B + s_2\cdot\sin\varphi_2\end{cases} \Rightarrow \\ \Rightarrow \begin{cases}\dot{x}_{G_2} = \dot{x}_B - s_2\cdot\sin\varphi_2\cdot\dot{\varphi}_2 \\ \dot{y}_{G_2} = \dot{y}_B + s_2\cdot\cos\varphi_2\cdot\dot{\varphi}_2\end{cases} \Rightarrow \\ \Rightarrow \begin{cases}\ddot{x}_{G_2} = \ddot{x}_B - s_2\cdot\cos\varphi_2\cdot\omega_2^2 - s_2\cdot\sin\varphi_2\cdot\varepsilon_2 \\ \ddot{y}_{G_2} = \ddot{y}_B - s_2\cdot\sin\varphi_2\cdot\omega_2^2 + s_2\cdot\cos\varphi_2\cdot\varepsilon_2\end{cases} \\ \begin{cases}x_{G_3} = x_D + s_{3'}\cdot\cos\varphi_{3'} \\ y_{G_3} = y_D + s_{3'}\cdot\sin\varphi_{3'}\end{cases} \Rightarrow \\ \Rightarrow \begin{cases}\dot{x}_{G_3} = \dot{x}_D - s_{3'}\cdot\sin\varphi_{3'}\cdot\dot{\varphi}_3 \\ \dot{y}_{G_3} = \dot{y}_D + s_{3'}\cdot\cos\varphi_{3'}\cdot\dot{\varphi}_3\end{cases} \Rightarrow \\ \Rightarrow \begin{cases}\ddot{x}_{G_3} = \ddot{x}_D - s_{3'}\cdot\cos\varphi_{3'}\cdot\omega_3^2 - s_{3'}\cdot\sin\varphi_{3'}\cdot\varepsilon_3 \\ \ddot{y}_{G_3} = \ddot{y}_D - s_{3'}\cdot\sin\varphi_{3'}\cdot\omega_3^2 + s_{3'}\cdot\cos\varphi_{3'}\cdot\varepsilon_3\end{cases}\end{cases} \quad (8)$$

3. DIAGRAMS OF THE FORCES FROM JOINTS

The joints forces can be determined and represented by the two diagrams below (Figure 2, and 3).

Below you can see the six forces (internal forces) of joints from dyad 3R, depending on the angle of the crank FI, when the dyad is linked together with a crank, forming a mechanism 4R [1-4].

Variation is represented on an entire cycle kinematic, for an angular velocity of crank, 200 or 300 [s^{-1}].

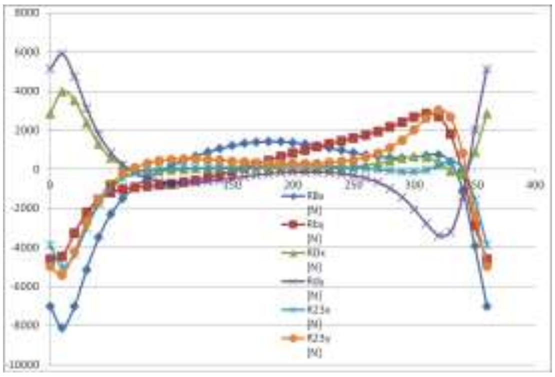

Fig. 2. *The six internal forces of joints;ω=200 [s^-1]*

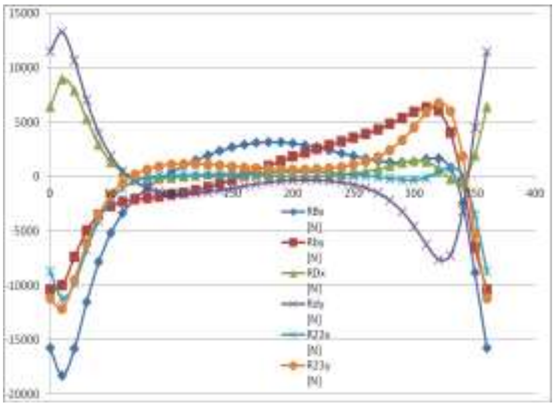

Fig. 3. *The six internal forces of joints;ω=300 [s^-1]*

4. CONCLUSIONS

This method presented in the article, is the most elegant and direct method to determine the internal forces at a 3R dyad [3] [1-11].

The method has a strong teaching character.

The relationships presented in this paper allow and the synthesis of robots (the mechanical systems, serial, in movement) [3].

5. IMPORTANCE AND USES

I-The first use of the reaction forces from couplings, is sizing of the kinematic couplings.

II-At the mechanisms with a degree of mobility, with the forces from driving coupling (R_B^x, R_B^y), it determines the required motor torque (M_m). We illustrate by the mechanism articulated quadrilateral (Fig. 4 and relationships 9).

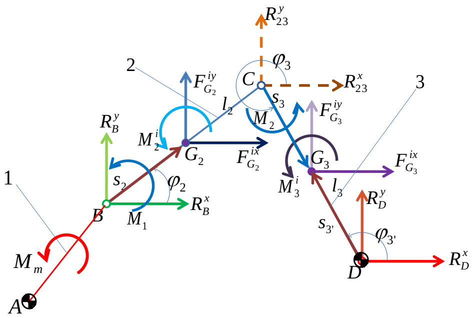

Fig. 4. *The forces at a mechanism articulated quadrilateral*

$$\begin{cases} M_m - R_{21}^x \cdot (y_B - y_A) + R_{21}^y \cdot (x_B - x_A) = 0 \Rightarrow \\ \Rightarrow M_m = R_{21}^x \cdot (y_B - y_A) - R_{21}^y \cdot (x_B - x_A) \Rightarrow \\ \Rightarrow M_m = -R_{12}^x \cdot (y_B - y_A) + R_{12}^y \cdot (x_B - x_A) \Rightarrow \\ \Rightarrow M_m = -R_B^x \cdot (y_B - y_A) + R_B^y \cdot (x_B - x_A) \end{cases} \quad (9)$$

Usually the torques M_1 and M_2 are null. But they can be and an external torque.

III-At the mechanisms with two degree of mobility, with the forces from driving coupling (see the Fig. 5), it determines the required motor torques: $M_1 \equiv M_{m2}, M_2 \equiv M_{m3}$.

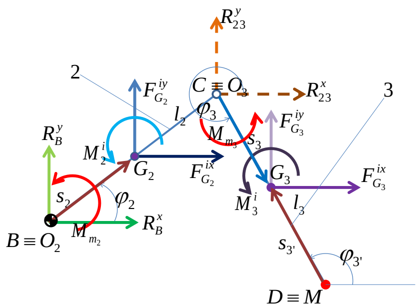

Fig. 5. *The forces at a mechanism with two degree of mobility*

This scheme is used in anthropomorphic robots. Coupling B is denoted by O_2. Coupling C is denoted by O_3. Coupling D become an end effector point M. Basic structure 3R of anthropomorphic robot (Fig. 6) can be decomposed into 2R planar structure (Fig. 5) which also possesses an additional rotating around a vertical axis (O_0O_1).

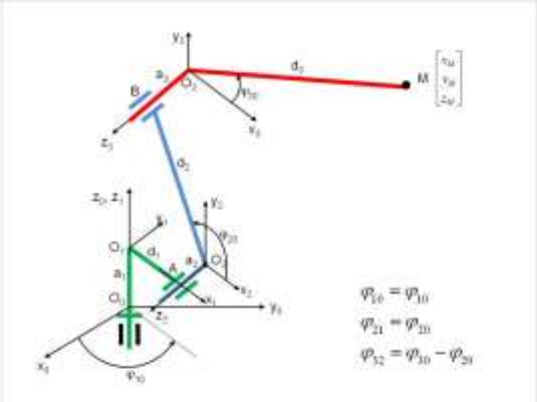

Fig. 6. *The basic structure 3R*

It is more convenient to study the structure plan O_2O_3M system (elements 2 and 3). But since this system (plan, 2R) using balanced, it's good to study in its balanced form (Fig. 7).

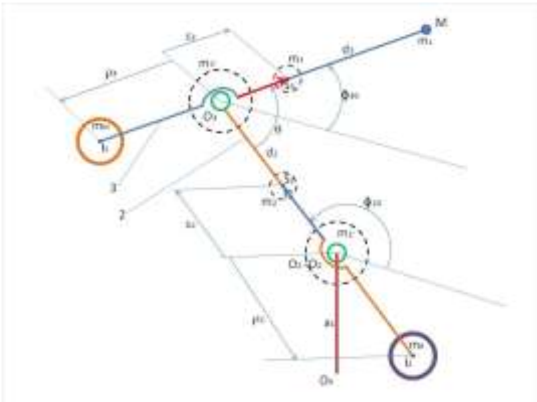

Fig. 7. *The basic, balanced, structure 2R*

Masses and lengths of the system are calculated using the equation 10.

$$\begin{cases} \begin{cases} \sum M_{O_3}^{(3)} = 0 \quad \Rightarrow \\ \Rightarrow m_s \cdot d_3 + m_3 \cdot s_3 = m_{III} \cdot \rho_3 \\ \Rightarrow \rho_3 = \dfrac{m_s \cdot d_3 + m_3 \cdot s_3}{m_{III}} \\ m_{3'} = m_3 + m_s + m_{III} \end{cases} \\ \begin{cases} \sum M_{O_2}^{(2+3)} = 0 \quad \Rightarrow \\ \Rightarrow m_{3'} \cdot d_2 + m_2 \cdot s_2 = m_{II} \cdot \rho_2 \\ \Rightarrow \rho_2 = \dfrac{m_{3'} \cdot d_2 + m_2 \cdot s_2}{m_{II}} \\ m_{2'} = m_{3'} + m_2 + m_{II} \end{cases} \end{cases} \tag{10}$$

Forces from the driveline balanced plan can be seen in the Fig. 8.

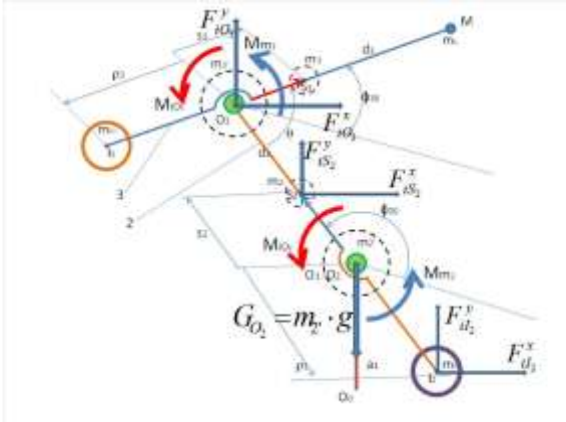

Fig. 8. *The forces of the basic (balanced) structure 2R*

Now, it still writing inertial forces (relations system 11) of the point O_3.

$$\begin{cases} F_{iO_3}^x = -m_{3'} \cdot \ddot{x}_{O_3} = \\ = -m_{3'} \cdot (-)d_2 \cdot \cos\varphi_{20} \cdot \omega_{20}^2 = \\ = m_{3'} \cdot d_2 \cdot \cos\varphi_{20} \cdot \omega_{20}^2 \\ \\ F_{iO_3}^y = -m_{3'} \cdot \ddot{y}_{O_3} = \\ = -m_{3'} \cdot (-)d_2 \cdot \sin\varphi_{20} \cdot \omega_{20}^2 = \\ = m_{3'} \cdot d_2 \cdot \sin\varphi_{20} \cdot \omega_{20}^2 \\ \\ M_{iO_3} = -J_{O_3} \cdot \varepsilon_3 \end{cases} \tag{11}$$

Now we are writing and the inertial forces of the points S_2 (12) and I_2 (13).

$$\begin{cases} F_{iS_2}^x = -m_2 \cdot \ddot{x}_{S_2} = m_2 \cdot s_2 \cdot \cos\varphi_{20} \cdot \omega_{20}^2 \\ F_{iS_2}^y = -m_2 \cdot \ddot{y}_{S_2} = m_2 \cdot s_2 \cdot \sin\varphi_{20} \cdot \omega_{20}^2 \end{cases} \tag{12}$$

$$\begin{cases} F_{iI_2}^x = -m_{II} \cdot \ddot{x}_{I_2} = -m_{II} \cdot \rho_2 \cdot \cos\varphi_{20} \cdot \omega_{20}^2 \\ F_{iI_2}^y = -m_{II} \cdot \ddot{y}_{I_2} = -m_{II} \cdot \rho_2 \cdot \sin\varphi_{20} \cdot \omega_{20}^2 \end{cases} \tag{13}$$

Now we can write the equilibrium equations on the element 2 projected on the x (system 14) and y (system 15).

$$\begin{cases} \sum F_{(2)}^x = 0 \Rightarrow m_{3'} \cdot d_2 \cdot \cos\varphi_{20} \cdot \omega_{20}^2 + m_2 \cdot s_2 \cdot \cos\varphi_{20} \cdot \omega_{20}^2 - \\ - m_{II} \cdot \rho_2 \cdot \cos\varphi_{20} \cdot \omega_{20}^2 + R_{O_2}^x = 0 \Rightarrow \\ \Rightarrow (m_{3'} \cdot d_2 + m_2 \cdot s_2 - m_{II} \cdot \rho_{II}) \cdot \cos\varphi_{20} \cdot \omega_{20}^2 + R_{12}^x = 0 \\ but \quad m_{3'} \cdot d_2 + m_2 \cdot s_2 - m_{II} \cdot \rho_{II} = 0 \quad \text{because balanced} \Rightarrow \\ \Rightarrow R_{O_2}^x \equiv R_{12}^x = 0 \end{cases} \tag{14}$$

$$\begin{cases} \sum F_{(2)}^y = 0 \Rightarrow m_{3'} \cdot d_2 \cdot \sin\varphi_{20} \cdot \omega_{20}^2 + m_2 \cdot s_2 \cdot \sin\varphi_{20} \cdot \omega_{20}^2 - \\ - m_{II} \cdot \rho_2 \cdot \sin\varphi_{20} \cdot \omega_{20}^2 - m_{2'} \cdot g + R_{12}^y = 0 \Rightarrow \\ \Rightarrow (m_{3'} \cdot d_2 + m_2 \cdot s_2 - m_{II} \cdot \rho_{II}) \cdot \sin\varphi_{20} \cdot \omega_{20}^2 - m_{2'} \cdot g + R_{12}^y = 0 \\ but \quad m_{3'} \cdot d_2 + m_2 \cdot s_2 - m_{II} \cdot \rho_{II} = 0 \quad \text{because balanced} \Rightarrow \\ \Rightarrow R_{O_2}^y \equiv R_{12}^y = m_{2'} \cdot g = G_{O_2} \end{cases} \tag{15}$$

It can be seen that the torque loads are minimal precisely because balancing. Effect given inertial forces (torques produced by these forces) cancel (balance due).

Torques produced by the forces of gravity is canceled and they all balance due.

Balanced final weight also makes the powertrain only one effect, a vertical load (causes a vertical reactor) in fixed coupling.

At a total balanced, even the horizontal load disappears.

It will still write an amount of moments to the fixed point O_2, on the element 2 (system 16).

$$\begin{cases} \sum M_{O_2}^{(2)} = 0 \Rightarrow M_{m_2} - F_{iO_3}^{x} \cdot d_2 \cdot \cos\left(\varphi_{20} - \frac{\pi}{2}\right) - \\ - F_{iO_3}^{y} \cdot d_2 \cdot \sin\left(\varphi_{20} - \frac{\pi}{2}\right) - F_{iS_2}^{x} \cdot s_2 \cdot \sin\varphi_{20} - F_{iS_2}^{y} \cdot s_2 \cdot -\cos\varphi_{20} + \\ + F_{iI_2}^{x} \cdot \rho_2 \cdot \cos\left(\varphi_{20} - \frac{\pi}{2}\right) + F_{iI_2}^{y} \cdot \rho_2 \cdot \sin\left(\varphi_{20} - \frac{\pi}{2}\right) + M_{iO_2} = 0 \Rightarrow \\ \\ \Rightarrow M_{m_2} - m_{3'} d_2^2 \omega_{20}^2 \cos\varphi_{20} \sin\varphi_{20} + m_{3'} \cdot d_2^2 \omega_{20}^2 \sin\varphi_{20} \cos\varphi_{20} - \\ - m_2 \cdot s_2^2 \cdot \omega_{20}^2 \cdot \cos\varphi_{20} \cdot \sin\varphi_{20} + m_2 \cdot s_2^2 \cdot \omega_{20}^2 \cdot \sin\varphi_{20} \cdot \cos\varphi_{20} - \\ - m_{II} \cdot \rho_2^2 \cdot \omega_{20}^2 \cos\varphi_{20} \cdot \sin\varphi_{20} + m_{II} \cdot \rho_2^2 \cdot \omega_{20}^2 \cdot \sin\varphi_{20} \cdot \cos\varphi_{20} - \\ - J_{O_2}^{*} \cdot \varepsilon_2 = 0 \Rightarrow M_{m_2} - J_{O_2}^{*} \cdot \varepsilon_2 = 0 \Rightarrow M_{m_2} = J_{O_2}^{*} \cdot \varepsilon_2 \end{cases} \tag{16}$$

Mass moment of inertia (or mechanical) of the element 2, is calculated with relation 17.

$$J_{O_2}^{*} = J_{O_2} + m_{3'} \cdot d_2^2 = m_2 \cdot s_2^2 + m_{II} \cdot \rho_2^2 + m_{3'} \cdot d_2^2 \tag{17}$$

One can determine now the torque required (M_{m2}), which must be generated by the actuator 2 (mounted in coupling O_2); see the relation (18).

$$M_{m_2} = J_{O_2}^{*} \cdot \varepsilon_2 = \left(m_2 \cdot s_2^2 + m_{II} \cdot \rho_2^2 + m_{3'} \cdot d_2^2\right) \cdot \ddot{\varphi}_{20} \tag{18}$$

We now sum of the moments of all forces on item 3 in relation to swivel O_3 (relationship 19).

$$\begin{aligned} &\sum M_{O_3}^{(3)} = 0 \Rightarrow \\ &M_{m_3} + M_{iO_3} = 0 \Rightarrow M_{m_3} - J_{O_3} \cdot \varepsilon_3 = 0 \Rightarrow \\ &\Rightarrow M_{m_3} = J_{O_3} \cdot \varepsilon_3 \Rightarrow \\ &\Rightarrow M_{m_3} = \left(m_s \cdot d_3^2 + m_3 \cdot s_3^2 + m_{III} \cdot \rho_3^2\right) \cdot \ddot{\varphi}_{30} \end{aligned} \tag{19}$$

One determines now and the vertical component, of the reaction, from the mobile (internal) coupling O_3; (see the relations of the system 20).

$$\begin{cases} \sum F_{(3)}^{y} = 0 \Rightarrow -m_{3'} \cdot g + R_{23}^{y} = 0 \Rightarrow \\ \Rightarrow R_{23}^{y} = m_{3'} \cdot g \Rightarrow \\ \Rightarrow R_{32}^{y} = -R_{23}^{y} = -m_{3'} \cdot g \end{cases} \tag{20}$$

Horizontal component (of the reaction from the kinematic coupling O_3) is zero (21).

$$R_{23}^{x} = -R_{32}^{y} = 0 \tag{21}$$

6. DYNAMICS OF SYSTEM 2R (LAGRANGE DIFFERENTIAL EQUATION OF THE SECOND KIND)

It writes now, just the most important relations of the system 2R, in the form 22.

$$\begin{cases} M_{m_2} = J_{O_2}^{*} \cdot \varepsilon_2 \\ M_{m_3} = J_{O_3} \cdot \varepsilon_3 \\ \\ M_{m_2} = \left(m_2 \cdot s_2^2 + m_{II} \cdot \rho_2^2 + m_{3'} \cdot d_2^2\right) \cdot \ddot{\varphi}_{20} \\ M_{m_3} = \left(m_s \cdot d_3^2 + m_3 \cdot s_3^2 + m_{III} \cdot \rho_3^2\right) \cdot \ddot{\varphi}_{30} \end{cases} \tag{22}$$

These relationships necessary to study the dynamics of the kinematic chain level (22), can be obtained directly by another method, which uses Lagrange differential equation of the second kind, and the kinetic energy saving mechanism.

This method is more direct than cinetostatic study, but has the disadvantage of not determining the loadings (reactions, internal forces) from kinematics chain, necessary to calculate the strength of the material in applications in which certain dimensions are selected (thickness or diameter) of the kinematic elements 2 and 3, and connecting joints.

One first determines the speeds, in the gravity centers (relations from system 23, and Fig. 9).

$$
\begin{cases}
\dot{x}_{O_2} = 0; \\
\dot{y}_{O_2} = 0; \quad \dot{\varphi}_{20} \equiv \omega_{20} \equiv \omega_2 \\
\dot{x}_{O_3} = -d_2 \cdot \sin\varphi_{20} \cdot \omega_2; \\
\dot{y}_{O_3} = d_2 \cdot \cos\varphi_{20} \cdot \omega_2; \\
\dot{\varphi}_{30} \equiv \omega_{30} \equiv \omega_3
\end{cases} \tag{23}
$$

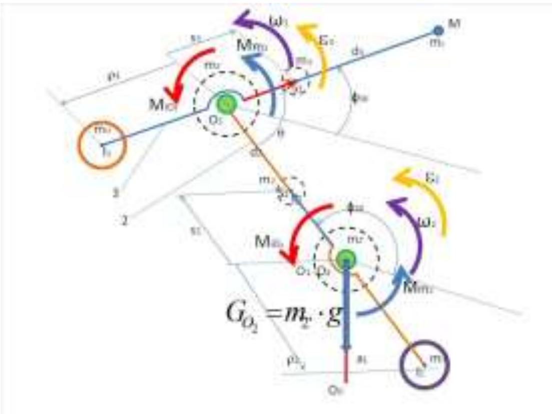

Fig. 9. *Dynamics of the driveline balanced plan*

For item 3, mass moment of inertia or mechanical (inertial mass) is determined by the relationship 24.

$$J_{O_3} = m_s \cdot d_3^2 + m_3 \cdot s_3^2 + m_{III} \cdot \rho_3^2 \tag{24}$$

For item 2, will cause mass moment of inertia (mechanical) in fixed joint O_2 (25).

$$J_{O_2} = m_2 \cdot s_2^2 + m_{II} \cdot \rho_2^2 \tag{25}$$

The kinetic energy of mechanism is determined with the relations of the system (26).

$$
\begin{cases}
E = \frac{1}{2} \cdot J_{O_2} \cdot \omega_2^2 + \frac{1}{2} \cdot J_{O_3} \cdot \omega_3^2 + \\
+ \frac{1}{2} \cdot m_{3'} \cdot \dot{x}_{O_3}^2 + \frac{1}{2} \cdot m_{3'} \cdot \dot{y}_{O_3}^2 = \\
= \frac{1}{2} \cdot J_{O_2} \cdot \omega_2^2 + \frac{1}{2} \cdot J_{O_3} \cdot \omega_3^2 + \\
+ \frac{1}{2} \cdot m_{3'} \cdot d_2^2 \cdot \omega_2^2 = \frac{1}{2} \cdot J_{O_3} \cdot \omega_3^2 + \\
+ \frac{1}{2} \cdot \omega_2^2 \cdot \left(J_{O_2} + m_{3'} \cdot d_2^2\right) = \\
= \frac{1}{2} \cdot J_{O_3} \cdot \omega_3^2 + \frac{1}{2} \cdot J_{O_2}^* \cdot \omega_2^2 \\
J_{O_2}^* = J_{O_2} + m_{3'} \cdot d_2^2
\end{cases} \tag{26}
$$

Kinetic energy equation for the balanced driveline is expressed with final relationship (27).

$$E = \frac{1}{2} \cdot J_{O_3} \cdot \omega_3^2 + \frac{1}{2} \cdot J_{O_2}^* \cdot \omega_2^2 \tag{27}$$

It uses the Lagrange differential equations of second kind (28).

$$
\begin{cases}
\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_k}\right) - \frac{\partial E}{\partial q_k} = Q_k \\
with \quad k = 2, \ 3 \\
\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_2}\right) - \frac{\partial E}{\partial q_2} = Q_2 \\
\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_3}\right) - \frac{\partial E}{\partial q_3} = Q_3
\end{cases} \tag{28}
$$

How kinetic energy in this case does not depend directly from the kinematic parameters of positions q_2 and q_3, represented by the position angles φ_{20} and φ_{30}, it can be used the simplified form of the Lagrange equations (29).

$$
\begin{cases}
\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_k}\right) = Q_k \quad with \quad k = 2, \ 3 \\
\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_2}\right) = Q_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial E}{\partial \omega_2}\right) = M_{m_2} \\
\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_3}\right) = Q_3 \Rightarrow \frac{d}{dt}\left(\frac{\partial E}{\partial \omega_3}\right) = M_{m_3}
\end{cases} \tag{29}
$$

By replacing the partial derivatives and making the derivatives in function of time, the system (29) takes the form (30).

$$
\begin{cases}
\frac{\partial E}{\partial \omega_2} = J_{O_2}^* \cdot \omega_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial E}{\partial \omega_2}\right) = J_{O_2}^* \cdot \varepsilon_2 \Rightarrow J_{O_2}^* \cdot \varepsilon_2 = M_{m_2} \\
\frac{\partial E}{\partial \omega_3} = J_{O_3} \cdot \omega_3 \Rightarrow \frac{d}{dt}\left(\frac{\partial E}{\partial \omega_3}\right) = J_{O_3} \cdot \varepsilon_3 \Rightarrow J_{O_3} \cdot \varepsilon_3 = M_{m_3} \\
J_{O_2}^* \cdot \varepsilon_2 = M_{m_2} \\
J_{O_3} \cdot \varepsilon_3 = M_{m_3} \\
J_{O_2}^* = m_2 \cdot s_2^2 + m_{II} \cdot \rho_2^2 + m_{3'} \cdot d_2^2 \\
J_{O_3} = m_s \cdot d_3^2 + m_3 \cdot s_3^2 + m_{III} \cdot \rho_3^2 \\
M_{m_2} = \left(m_2 \cdot s_2^2 + m_{II} \cdot \rho_2^2 + m_{3'} \cdot d_2^2\right) \cdot \varepsilon_2 \\
M_{m_3} = \left(m_s \cdot d_3^2 + m_3 \cdot s_3^2 + m_{III} \cdot \rho_3^2\right) \cdot \varepsilon_3
\end{cases} \tag{30}
$$

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