ISSN: 1415-7314
ISSN online: 2317-6717

ENGEVISTA
Página da revista: http://www.uff.br/engevista/seer/

Velocities and accelerations at the 3r robots

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Abstract: The paper presents an original method to determine the velocities and the accelerations at the MP-3R structures. At the 3R structure (spatial) are known (imposed) the angular speeds of actuators and must be determined the velocities and the accelerations of the endeffector point M. Starting from the MP-3R direct kinematic positions system, deriving these relations system in function of the time, one time and then a second time (the second derivation) one obtains first the system velocities, and second time the accelerations of the point endeffector M. The system which must be solved has three equations and three independent parameters to determine. Constructive basis is represented by a robot with three degrees of freedom (a robot with three axes of rotation). If one study (analyzes) an anthropomorphic robot with three axes of rotation (which represents the main movements, absolutely necessary), it already has a base system, on which one can then add other movements (secondary, additional). All calculations were arranged and in the matrix form.

Keywords: **A**nthropomorphic robots, direct kinematics, 3R systems, matrix systems, velocities, accelerations.

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ISSN: 1415-7314
ISSN online: 2317-6717

1. Introduction

Although the anthropomorphic robots, have different structural forms, in recent years have been developed especially those with rotating movements, with three or more axis. Constructive basis is represented by a robot with three degrees of freedom (a robot with three axes of rotation) [1]. If one study (analyze) an anthropomorphic robot with three axes of rotation (which represents the main movements, absolutely necessary), we already have a base system, on which we can then add other movements (secondary, additional). The base system has three rotary axes: a vertical axis (by this axis all the system is rotated, for positioning), and two horizontal axes (each making possible a rotation of an arm). Calculations were arranged and in the matrix form.

In direct kinematics are known the kinematic parameters (input parameters) which are the absolute rotation angles of the three mobile elements: φ_{10}, φ_{20}, φ_{30}, the rotation angles of the three actuators (electric motors, mounted in the rotational kinematic couplings), and the determined parameters (the output parameters) are the three absolute coordinates x_M, y_M, z_M of the point M, ie kinematic parameters (coordinates) of the endeffector (which can be a hand, to grabbed, a soldering tip, painted, cut, etc).

In inverse kinematics [2-8], one already knows the coordinates x_M, y_M, z_M of the point M, and must be determined the independent rotations φ_{10}, φ_{20}, φ_{30} of the three mobile elements, based on kinematic parameters imposed to the endeffector x_M, y_M, z_M, known (forced).

With the independent determined angles, is then to be calculated the relative rotation movements, of the three driving motors, from the rotating couplers [7].

Considering the positions already determined, it imposes the problem of determining the velocities and accelerations of the system.

2. Determining the positions at the 3R robots (systems)

Kinematics of serial manipulators and robots will be illustrated by a 3R kinematic model (see Fig. 1), a medium difficulty system, ideal for understanding the phenomenon, but also to specify the basic knowledge necessary for starting calculations for systems simpler and more complex.

ISSN: 1415-7314
ISSN online: 2317-6717

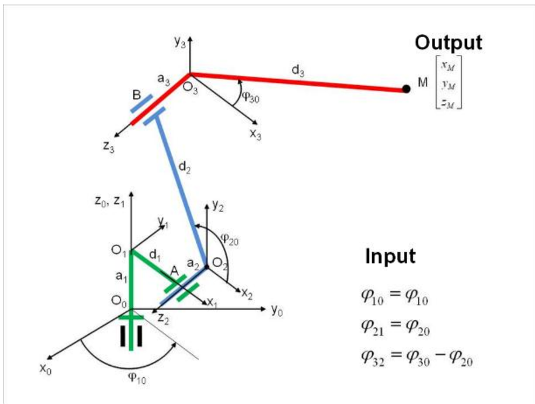

Figure 1. Geometry and direct kinematics to a MP-3R

Fixed coordinate system was noted with $x_0O_0y_0z_0$. Mobile systems related to (reinforced by) the three mobile elements (1, 2, 3) have indices 1, 2 and 3. Their orientation was chosen conveniently. Known kinematic parameters (input parameters in direct kinematics) are absolute rotation angles of the three mobile elements: φ_{10}, φ_{20}, φ_{30}, the rotation angles of the three actuators (electric motors, mounted in the rotational kinematic couplings). Determined parameters (output parameters) are the three absolute coordinates x_M, y_M, z_M of the point M, ie kinematic parameters (coordinates) of the endeffector (which can be a hand, to grabbed, a soldering tip, painted, cut, etc).

To begin one writes vector matrix (A_{01}) which change the coordinates of the origin of the coordinate system, by linear moving (displacement) from O_0 to O_1, when the axes remain parallel to each other permanently (see Eq. 2.1).

$$A_{01} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \qquad (2.1)$$

Next we write the rotation matrix T_{01}, which rotates system $x_1O_1y_1z_1$ in rapport with the system $x_0O_0y_0z_0$ (it is a 3x3 square matrix; see the relationship 2.2).

$$T_{01} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.2)$$

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On the first column (which represents the coordinates of the rotated axis O_1x_1) it writes the coordinates of the unit vector of O_1x_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O1 but without rotation; see the relationship 2.3).

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \qquad (2.3)$$

On the second column of the matrix T_{01} it writes the coordinates of the unit vector of the rotated axis O_1y_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O_1 but without rotation system; see the relationship 2.4).

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} \qquad (2.4)$$

On the third column of the matrix T_{01} it writes the coordinates of the unit vector of the rotated axis O_1z_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O_1 but without rotation system; see the relationship 2.5).

$$\begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \end{bmatrix} \qquad (2.5)$$

In the elected case (figure 1), the unit vector of the rotated axis O_1x_1, has in rapport of the old system $x_0O_0y_0z_0$, translated into O_1 without rotation, the coordinates given by the column unit vector (relationship 2.6).

$$\begin{bmatrix} \alpha_x = 1 \cdot \cos\varphi_{10} = \cos\varphi_{10} \\ \alpha_y = 1 \cdot \sin\varphi_{10} = \sin\varphi_{10} \\ \alpha_z = 1 \cdot \cos 90^0 = 1 \cdot 0 = 0 \end{bmatrix} \qquad (2.6)$$

The unit vector of the rotated axis O_1y_1, has in rapport of the old system of axes $x_0O_0y_0z_0$ (translated into O1 without rotation), coordinates data unit vector column (relationship 2.7).

$$\begin{bmatrix} \beta_x = 1 \cdot \cos(\pi/2 + \varphi_{10}) = -\sin\varphi_{10} \\ \beta_y = 1 \cdot \sin(\pi/2 + \varphi_{10}) = \cos\varphi_{10} \\ \beta_z = 1 \cdot \cos(\pi/2) = 1 \cdot 0 = 0 \end{bmatrix} \qquad (2.7)$$

The unit vector of the rotated axis O_1z_1 has in rapport of the old system of axes $x_0O_0y_0z_0$ (translated into O_1 without rotation), coordinates data unit vector column (relationship 2.8).

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$$\begin{bmatrix} \gamma_x = 1 \cdot \cos 90^0 = 1 \cdot 0 = 0 \\ \gamma_y = 1 \cdot \cos 90^0 = 1 \cdot 0 = 0 \\ \gamma_z = 1 \cdot \cos 0^0 = 1 \cdot 1 = 1 \end{bmatrix} \quad (2.8)$$

See the obtained matrix T_{01} (relationship 2.2).

Transition from the coordinate system $x_1O_1y_1z_1$ to the coordinate system $x_2O_2y_2z_2$ is done in two distinct phases. The first phase is a translation of the entire system so that (axes being parallel with them itself) the center O_1 to move into the center O_2; then the second stage in which it done the rotation of system of axes, and the center O remains fixed permanently.

The translation of the system from point 1 to the point 2 (see the relationship 2.9) is doing by the column vector, matrix A_{12}.

$$A_{12} = \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} \quad (2.9)$$

On the old O1x1 axis O2 has been moved with d1, on the old axis O1y1 O2 has been moved with a2, and on the old O1z1 axis O2 has not been moved.

The unit vector of the O_2x_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated but not rotated) the next coordinates (expression 2.10).

$$\alpha_x = 1; \quad \alpha_y = 0; \quad \alpha_z = 0 \quad (2.10)$$

The unit vector of the O_2y_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated in O_2 but not rotated) the next coordinates (expression 2.11).

$$\beta_x = 0; \quad \beta_y = 0; \quad \beta_z = 1 \quad (2.11)$$

The unit vector of the O_2z_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated in O_2 but not rotated) the coordinates given by the expression 2.12.

$$\gamma_x = 0; \quad \gamma_y = -1; \quad \gamma_z = 0 \quad (2.12)$$

The transfer square matrix (the rotation matrix: T12) is writing with relationship 2.13.

$$T_{12} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad (2.13)$$

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Transition from the coordinate system $x_2O_2y_2z_2$ to the coordinate system $x_3O_3y_3z_3$ is done in two distinct phases. The first phase is a translation of the entire system so that (axes being parallel with them itself) the center O_2 to move into the center O_3; then the second stage in which it done the rotation of system of axes, and the center O_3 remains fixed permanently.

First O_2 is moving into O_3 (axes being parallel with them itself; see the relationship 2.14).

$$A_{23} = \begin{bmatrix} d_2 \cdot \cos\varphi_{20} \\ d_2 \cdot \sin\varphi_{20} \\ -a_3 \end{bmatrix} \qquad (2.14)$$

Then O_3 remains fixed, and the axes of coordinate system are rotating. The unit vector of the O_3x_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the α coordinates (see expression 2.15):

$$\alpha_x = 1; \quad \alpha_y = 0; \quad \alpha_z = 0 \qquad (2.15)$$

The unit vector of the O_3y_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the β coordinates (see relationship 2.16):

$$\beta_x = 0; \quad \beta_y = 1; \quad \beta_z = 0 \qquad (2.16)$$

The unit vector of the O_3z_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the γ coordinates (see relationship 2.17):

$$\gamma_x = 0; \quad \gamma_y = 0; \quad \gamma_z = 1 \qquad (2.17)$$

In the model from the figure 1 the system $x_3O_3y_3z_3$ has not been rotated in rapport of the system $x_2O_2y_2z_2$ (from 2 to 3 held just a translation). In this case the rotation matrix is the unit matrix (expression 2.18).

$$T_{23} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.18)$$

The column vector matrix that positions the point M in the coordinate system $x_3O_3y_3z_3$ is written with relation 2.19.

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$$X_{3M}=\begin{bmatrix} x_{3M} \\ y_{3M} \\ z_{3M} \end{bmatrix}=\begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} \qquad (2.19)$$

Coordinates of the point M in the system (2) $x_2O_2y_2z_2$ are obtained by a transformation matrix which is having the form (2.20):

$$X_{2M}=A_{23}+T_{23}\cdot X_{3M} \qquad (2.20)$$

First, is performed the matrix product (relations 2.21):

$$T_{23}\cdot X_{3M}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix}=\begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} \qquad (2.21)$$

Then, will be calculated X_{2M} (relationship 2.22).

$$X_{2M}=A_{23}+T_{23}\cdot X_{3M}=\begin{bmatrix} d_2 \cdot \cos\varphi_{20} \\ d_2 \cdot \sin\varphi_{20} \\ -a_3 \end{bmatrix}+\begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix}=\begin{bmatrix} d_2 \cdot \cos\varphi_{20}+d_3 \cdot \cos\varphi_{30} \\ d_2 \cdot \sin\varphi_{20}+d_3 \cdot \sin\varphi_{30} \\ -a_3 \end{bmatrix} \qquad (2.22)$$

Coordinates of the point M in the system (1) $x_1O_1y_1z_1$ are obtained by the relationships (2.23-2.25).

$$X_{1M}=A_{12}+T_{12}\cdot X_{2M} \qquad (2.23)$$

$$T_{12}\cdot X_{2M}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\cdot\begin{bmatrix} d_2 \cdot \cos\varphi_{20}+d_3 \cdot \cos\varphi_{30} \\ d_2 \cdot \sin\varphi_{20}+d_3 \cdot \sin\varphi_{30} \\ -a_3 \end{bmatrix}=\begin{bmatrix} d_2 \cdot \cos\varphi_{20}+d_3 \cdot \cos\varphi_{30} \\ a_3 \\ d_2 \cdot \sin\varphi_{20}+d_3 \cdot \sin\varphi_{30} \end{bmatrix} \qquad (2.24)$$

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$$X_{1M} = A_{12} + T_{12} \cdot X_{2M} = \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} + \begin{bmatrix} d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30} \\ a_3 \\ d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix} = \begin{bmatrix} d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30} \\ a_2 + a_3 \\ d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix} \quad (2.25)$$

Coordinates of the point M in the fixed system $x_0O_0y_0z_0$, are written with the relationships (2.26-2.27, 2.27', 2.28).

$$X_{0M} = A_{01} + T_{01} \cdot X_{1M} \qquad (2.26)$$

$$T_{01} \cdot X_{1M} = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30} \\ a_2 + a_3 \\ d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix} \qquad (2.27)$$

$$T_{01} \cdot X_{1M} = \begin{bmatrix} (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \cos\varphi_{10} - (a_2 + a_3) \cdot \sin\varphi_{10} \\ (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \sin\varphi_{10} + (a_2 + a_3) \cdot \cos\varphi_{10} \\ d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix} \qquad (2.27')$$

$$X_{0M} = A_{01} + T_{01} \cdot X_{1M} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} + \begin{bmatrix} (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \cos\varphi_{10} - (a_2 + a_3) \cdot \sin\varphi_{10} \\ (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \sin\varphi_{10} + (a_2 + a_3) \cdot \cos\varphi_{10} \\ d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix} = \begin{bmatrix} (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \cos\varphi_{10} - (a_2 + a_3) \cdot \sin\varphi_{10} \\ (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \sin\varphi_{10} + (a_2 + a_3) \cdot \cos\varphi_{10} \\ a_1 + d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix}$$

(2.28)

X_{0M} is arranged in the form (2.29).

$$X_{0M} = \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} = \begin{bmatrix} d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \cos\varphi_{10} - a_3 \cdot \sin\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \cos\varphi_{10} \\ d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \sin\varphi_{10} + a_3 \cdot \cos\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \sin\varphi_{10} \\ a_1 + d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix} \qquad (2.29)$$

The same calculations will be presented now by a direct method (having in view the matrix calculations 2.30).

$$\begin{aligned} X_{0M} &= A_{01} + T_{01} \cdot X_{1M} = A_{01} + T_{01} \cdot (A_{12} + T_{12} \cdot X_{2M}) = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot X_{2M} = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot (A_{23} + T_{23} \cdot X_{3M}) = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \end{aligned} \qquad (2.30)$$

It keeps the relationship (2.30').

$$X_{0M} = A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \qquad (2.30')$$

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ISSN online: 2317-6717

Now, one performs the matrix multiplications from expression 2.30’ (relationships 2.31-2.35).

$$T_{01} \cdot A_{12} = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} = \begin{bmatrix} d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} \\ d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} \\ 0 \end{bmatrix} \tag{2.31}$$

$$T_{01} \cdot T_{12} = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \tag{2.32}$$

$$T_{01} \cdot T_{12} \cdot A_{23} =$$
$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_2 \cdot \cos\varphi_{20} \\ d_2 \cdot \sin\varphi_{20} \\ -a_3 \end{bmatrix} = \begin{bmatrix} d_2 \cdot \cos\varphi_{10} \cdot \cos\varphi_{20} - a_3 \cdot \sin\varphi_{10} \\ d_2 \cdot \sin\varphi_{10} \cdot \cos\varphi_{20} + a_3 \cdot \cos\varphi_{10} \\ d_2 \cdot \sin\varphi_{20} \end{bmatrix} \tag{2.33}$$

$$T_{01} \cdot T_{12} \cdot T_{23} =$$
$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \tag{2.34}$$

$$T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} =$$
$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\varphi_{10} \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{10} \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \end{bmatrix} \tag{2.35}$$

The expression (2.30’) takes the form (2.36).

$$X_{0M} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} + \begin{bmatrix} d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} \\ d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} \\ 0 \end{bmatrix} + \begin{bmatrix} d_2 \cdot \cos\varphi_{10} \cdot \cos\varphi_{20} - a_3 \cdot \sin\varphi_{10} \\ d_2 \cdot \sin\varphi_{10} \cdot \cos\varphi_{20} + a_3 \cdot \cos\varphi_{10} \\ d_2 \cdot \sin\varphi_{20} \end{bmatrix} + \tag{2.36}$$
$$+ \begin{bmatrix} d_3 \cdot \cos\varphi_{10} \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{10} \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \end{bmatrix} = \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} = \begin{bmatrix} d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \cos\varphi_{10} - a_3 \cdot \sin\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \cos\varphi_{10} \\ d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \sin\varphi_{10} + a_3 \cdot \cos\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \sin\varphi_{10} \\ a_1 + d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix}$$

By the direct kinematics is obtained Cartesian coordinates x_M, y_M, z_M of the point M (the endeffector) in rapport with the three independent angular displacements φ_{10}, φ_{20}, φ_{30}, obtained using actuators (relationships 2.37-2.38).

$$\begin{cases} x_M = f_x(\varphi_{10}, \ \varphi_{20}, \ \varphi_{30}) \\ y_M = f_y(\varphi_{10}, \ \varphi_{20}, \ \varphi_{30}) \\ z_M = f_z(\varphi_{10}, \ \varphi_{20}, \ \varphi_{30}) \end{cases} \qquad (2.37)$$

$$\begin{cases} x_M = d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \cos\varphi_{10} - a_3 \cdot \sin\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \cos\varphi_{10} \\ y_M = d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \sin\varphi_{10} + a_3 \cdot \cos\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \sin\varphi_{10} \\ z_M = a_1 + d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{cases}$$

(2.38)

Calculations are performed with absolute angular movements (φ_{10}, φ_{20}, φ_{30}), but the actuators movements do not match (all) with the independent angular movements. They are determined as follows (expressions 2.39):

$$\begin{cases} \varphi_{10} = \varphi_{10} \\ \varphi_{21} = \varphi_{20} \\ \varphi_{32} = \varphi_{30} - \varphi_{20} \end{cases} \qquad (2.39)$$

The first two actuators relative rotations coincide with the independent rotations (used in calculations), but the third actuator relative rotation is obtained as a difference between two absolute rotations (expressions 2.39). The velocities and the accelerations are obtained by the derivatives of the positions expressions (2.38) in rapport of the time.

3. Determining the velocities and the accelerations at the 3r robots (systems)

It starts from the relationship matrix gear (3.1) already known.

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$$X_{0M} = A_{01} + T_{01} \cdot X_{1M} = A_{01} + T_{01} \cdot (A_{12} + T_{12} \cdot X_{2M}) = \\ = A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot X_{2M} = \\ = A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot (A_{23} + T_{23} \cdot X_{3M}) = \\ = A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \quad (3.1)$$

This is written as (3.2) simplified:

$$X_{0M} = A_{01} + P_1 + P_2 + T_{03} \cdot X_{3M} \quad (3.2)$$

Where:

$$A_{01} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \quad (3.3)$$

$$P_1 = \begin{bmatrix} d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} \\ d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} \\ 0 \end{bmatrix} \quad (3.4)$$

$$P_2 = \begin{bmatrix} d_2 \cdot \cos\varphi_{10} \cdot \cos\varphi_{20} - a_3 \cdot \sin\varphi_{10} \\ d_2 \cdot \sin\varphi_{10} \cdot \cos\varphi_{20} + a_3 \cdot \cos\varphi_{10} \\ d_2 \cdot \sin\varphi_{20} \end{bmatrix} \quad (3.5)$$

$$T_{03} = \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \quad (3.6)$$

$$X_{3M} = \begin{bmatrix} x_{3M} \\ y_{3M} \\ z_{3M} \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} \quad (3.7)$$

The Velocities

It derives the relationship (3.2) a matrix and obtain expression (3.8):

$$\dot{X}_{0M} = \dot{A}_{01} + \dot{P}_1 + \dot{P}_2 + \dot{T}_{03} \cdot X_{3M} + T_{03} \cdot \dot{X}_{3M} = \dot{P}_1 + \dot{P}_2 + \dot{T}_{03} \cdot X_{3M} + T_{03} \cdot \dot{X}_{3M} = \dot{P}_{12} + \dot{T}_{03} \cdot X_{3M} + T_{03} \cdot \dot{X}_{3M} \quad (3.8)$$

Seeing that:

$$\dot{A}_{01} = \begin{bmatrix} 0 \\ 0 \\ \dot{a}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \quad (3.9)$$

ISSN: 1415-7314
ISSN online: 2317-6717

$$\dot{P}_1 = \begin{bmatrix} -d_1 \cdot \sin\varphi_{10} \cdot \omega_{10} - a_2 \cdot \cos\varphi_{10} \cdot \omega_{10} \\ d_1 \cdot \cos\varphi_{10} \cdot \omega_{10} - a_2 \cdot \sin\varphi_{10} \cdot \omega_{10} \\ 0 \end{bmatrix} \quad (3.10)$$

$$\dot{P}_2 = \begin{bmatrix} -d_2 \cdot \sin\varphi_{10} \cdot \omega_{10} \cdot \cos\varphi_{20} - d_2 \cdot \cos\varphi_{10} \cdot \sin\varphi_{20} \cdot \omega_{20} - a_3 \cdot \cos\varphi_{10} \cdot \omega_{10} \\ d_2 \cdot \cos\varphi_{10} \cdot \omega_{10} \cdot \cos\varphi_{20} - d_2 \cdot \sin\varphi_{10} \cdot \sin\varphi_{20} \cdot \omega_{20} - a_3 \cdot \sin\varphi_{10} \cdot \omega_{10} \\ d_2 \cdot \cos\varphi_{20} \cdot \omega_{20} \end{bmatrix} \quad (3.11)$$

$$\dot{T}_{03} = \begin{bmatrix} -\sin\varphi_{10} \cdot \omega_{10} & 0 & \cos\varphi_{10} \cdot \omega_{10} \\ \cos\varphi_{10} \cdot \omega_{10} & 0 & \sin\varphi_{10} \cdot \omega_{10} \\ 0 & 0 & 0 \end{bmatrix} \quad (3.12)$$

$$\dot{X}_{3M} = \begin{bmatrix} \dot{x}_{3M} \\ \dot{y}_{3M} \\ \dot{z}_{3M} \end{bmatrix} = \begin{bmatrix} -d_3 \cdot \sin\varphi_{30} \cdot \omega_{30} \\ d_3 \cdot \cos\varphi_{30} \cdot \omega_{30} \\ 0 \end{bmatrix} \quad (3.13)$$

$$\dot{P}_{12} = \dot{P}_1 + \dot{P}_2 = \begin{bmatrix} -d_1 \sin\varphi_{10}\omega_{10} - a_2 \cos\varphi_{10}\omega_{10} - a_3 \cos\varphi_{10}\omega_{10} - d_2 \sin\varphi_{10}\omega_{10} \cos\varphi_{20} - d_2 \cos\varphi_{10} \sin\varphi_{20}\omega_{20} \\ d_1 \cos\varphi_{10}\omega_{10} - a_2 \sin\varphi_{10}\omega_{10} - a_3 \sin\varphi_{10}\omega_{10} + d_2 \cos\varphi_{10}\omega_{10} \cos\varphi_{20} - d_2 \sin\varphi_{10} \sin\varphi_{20}\omega_{20} \\ d_2 \cos\varphi_{20}\omega_{20} \end{bmatrix} \quad (3.14)$$

The following two products is determined: matrix (3.15 and 3.16), from equation (3.8).

$$\dot{T}_{03} \cdot X_{3M} = \begin{bmatrix} -\sin\varphi_{10} \cdot \omega_{10} & 0 & \cos\varphi_{10} \cdot \omega_{10} \\ \cos\varphi_{10} \cdot \omega_{10} & 0 & \sin\varphi_{10} \cdot \omega_{10} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} = \begin{bmatrix} -d_3 \cdot \sin\varphi_{10} \cdot \omega_{10} \cdot \cos\varphi_{30} \\ d_3 \cdot \cos\varphi_{10} \cdot \omega_{10} \cdot \cos\varphi_{30} \\ 0 \end{bmatrix} \quad (3.15)$$

$$T_{03}\cdot \dot{X}_{3M}=\begin{bmatrix}\cos\varphi_{10} & 0 & \sin\varphi_{10}\\ \sin\varphi_{10} & 0 & -\cos\varphi_{10}\\ 0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}-d_3\cdot\sin\varphi_{30}\cdot\omega_{30}\\ d_3\cdot\cos\varphi_{30}\cdot\omega_{30}\\ 0\end{bmatrix}=\begin{bmatrix}-d_3\cdot\cos\varphi_{10}\cdot\sin\varphi_{30}\cdot\omega_{30}\\ -d_3\cdot\sin\varphi_{10}\cdot\sin\varphi_{30}\cdot\omega_{30}\\ d_3\cdot\cos\varphi_{30}\cdot\omega_{30}\end{bmatrix} \quad (3.16)$$

One can determine now $\dot{X}_{0M}$ (system 3.17):

$$\dot{X}_{0M}=\begin{bmatrix}(-d_1\sin\varphi_{10}\omega_{10}-a_2\cos\varphi_{10}\omega_{10}-a_3\cos\varphi_{10}\omega_{10}-d_2\sin\varphi_{10}\omega_{10}\cos\varphi_{20}-\\ -d_2\cos\varphi_{10}\sin\varphi_{20}\omega_{20}-d_3\sin\varphi_{10}\omega_{10}\cos\varphi_{30}-d_3\cos\varphi_{10}\sin\varphi_{30}\omega_{30})\\ (d_1\cos\varphi_{10}\omega_{10}-a_2\sin\varphi_{10}\omega_{10}-a_3\sin\varphi_{10}\omega_{10}+d_2\cos\varphi_{10}\omega_{10}\cos\varphi_{20}-\\ -d_2\sin\varphi_{10}\sin\varphi_{20}\omega_{20}+d_3\cos\varphi_{10}\omega_{10}\cos\varphi_{30}-d_3\sin\varphi_{10}\sin\varphi_{30}\omega_{30})\\ (d_2\cos\varphi_{20}\omega_{20}+d_3\cos\varphi_{30}\omega_{30})\end{bmatrix} \quad (3.17)$$

The Accelerations

Follow relations accelerations. It derives the relation (3.8) to give the expression (3.18):

$$\ddot{X}_{0M}=\ddot{P}_{12}+\ddot{T}_{03}\cdot X_{3M}+\dot{T}_{03}\cdot\dot{X}_{3M}+\dot{T}_{03}\cdot\dot{X}_{3M}+T_{03}\cdot\ddot{X}_{3M}=\ddot{P}_{12}+\ddot{T}_{03}\cdot X_{3M}+2\cdot\dot{T}_{03}\cdot\dot{X}_{3M}+T_{03}\cdot\ddot{X}_{3M} \quad (3.18)$$

$$\ddot{P}_{12}=\ddot{P}_1+\ddot{P}_2=$$

$$=\begin{bmatrix}(-d_1\cos\varphi_{10}\omega_{10}^2+a_2\sin\varphi_{10}\omega_{10}^2+a_3\sin\varphi_{10}\omega_{10}^2-d_2\cos\varphi_{10}\omega_{10}^2\cos\varphi_{20}+\\ +d_2\sin\varphi_{10}\omega_{10}\sin\varphi_{20}\omega_{20}+d_2\sin\varphi_{10}\omega_{10}\sin\varphi_{20}\omega_{20}-d_2\cos\varphi_{10}\cos\varphi_{20}\omega_{20}^2)\\ (-d_1\sin\varphi_{10}\omega_{10}^2-a_2\cos\varphi_{10}\omega_{10}^2-a_3\cos\varphi_{10}\omega_{10}^2-d_2\sin\varphi_{10}\omega_{10}^2\cos\varphi_{20}-\\ -d_2\cos\varphi_{10}\omega_{10}\sin\varphi_{20}\omega_{20}-d_2\cos\varphi_{10}\omega_{10}\sin\varphi_{20}\omega_{20}-d_2\sin\varphi_{10}\cos\varphi_{20}\omega_{20}^2)\\ (-d_2\sin\varphi_{20}\omega_{20}^2)\end{bmatrix} \quad (3.19)$$

$$\ddot{T}_{03}=\begin{bmatrix}-\cos\varphi_{10}\cdot\omega_{10}^2 & 0 & -\sin\varphi_{10}\cdot\omega_{10}^2\\ -\sin\varphi_{10}\cdot\omega_{10}^2 & 0 & \cos\varphi_{10}\cdot\omega_{10}^2\\ 0 & 0 & 0\end{bmatrix} \quad (3.20)$$

$$\ddot{X}_{3M}=\begin{bmatrix}-d_3\cdot\cos\varphi_{30}\cdot\omega_{30}^2\\ -d_3\cdot\sin\varphi_{30}\cdot\omega_{30}^2\\ 0\end{bmatrix} \quad (3.21)$$

ISSN: 1415-7314
ISSN online: 2317-6717

$$2\cdot\dot{T}_{03}\cdot\dot{X}_{3M}=\begin{bmatrix}2\cdot d_3\cdot\sin\varphi_{10}\cdot\omega_{10}\cdot\sin\varphi_{30}\cdot\omega_{30}\\-2\cdot d_3\cdot\cos\varphi_{10}\cdot\omega_{10}\cdot\sin\varphi_{30}\cdot\omega_{30}\\0\end{bmatrix}\qquad(3.22)$$

$$\ddot{T}_{03}\cdot X_{3M}=\begin{bmatrix}-\cos\varphi_{10}\cdot\omega_{10}^2 & 0 & -\sin\varphi_{10}\cdot\omega_{10}^2\\-\sin\varphi_{10}\cdot\omega_{10}^2 & 0 & \cos\varphi_{10}\cdot\omega_{10}^2\\0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}d_3\cdot\cos\varphi_{30}\\d_3\cdot\sin\varphi_{30}\\0\end{bmatrix}=\begin{bmatrix}-d_3\cdot\cos\varphi_{10}\cdot\omega_{10}^2\cdot\cos\varphi_{30}\\-d_3\cdot\sin\varphi_{10}\cdot\omega_{10}^2\cdot\cos\varphi_{30}\\0\end{bmatrix}\qquad(3.23)$$

$$T_{03}\cdot\ddot{X}_{3M}=\begin{bmatrix}\cos\varphi_{10} & 0 & \sin\varphi_{10}\\\sin\varphi_{10} & 0 & -\cos\varphi_{10}\\0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}-d_3\cdot\cos\varphi_{30}\cdot\omega_{30}^2\\-d_3\cdot\sin\varphi_{30}\cdot\omega_{30}^2\\0\end{bmatrix}=\begin{bmatrix}-d_3\cdot\cos\varphi_{10}\cdot\cos\varphi_{30}\cdot\omega_{30}^2\\-d_3\cdot\sin\varphi_{10}\cdot\cos\varphi_{30}\cdot\omega_{30}^2\\-d_3\cdot\sin\varphi_{30}\cdot\omega_{30}^2\end{bmatrix}\qquad(3.24)$$

It obtains the matrix of the endeffector accelerations (3.25) in function of the three actuators rotations (angular positions and velocities). With $\omega_{10}=ct,\ \ \omega_{20}=ct,\ \ \omega_{30}=ct$.

$$\ddot{X}_{0M}=$$
$$\begin{bmatrix}(-d_1\cos\varphi_{10}\omega_{10}^2+a_2\sin\varphi_{10}\omega_{10}^2+a_3\sin\varphi_{10}\omega_{10}^2-\\-d_2\cos\varphi_{10}\omega_{10}^2\cos\varphi_{20}+2d_2\sin\varphi_{10}\omega_{10}\sin\varphi_{20}\omega_{20}-\\-d_2\cos\varphi_{10}\cos\varphi_{20}\omega_{20}^2+2d_3\sin\varphi_{10}\omega_{10}\sin\varphi_{30}\omega_{30}-\\-d_3\cos\varphi_{10}\omega_{10}^2\cos\varphi_{30}-d_3\cos\varphi_{10}\cos\varphi_{30}\omega_{30}^2)\\ \\(-d_1\sin\varphi_{10}\omega_{10}^2-a_2\cos\varphi_{10}\omega_{10}^2-a_3\cos\varphi_{10}\omega_{10}^2-\\-d_2\sin\varphi_{10}\omega_{10}^2\cos\varphi_{20}-2d_2\cos\varphi_{10}\omega_{10}\sin\varphi_{20}\omega_{20}-\\-d_2\sin\varphi_{10}\cos\varphi_{20}\omega_{20}^2-2d_3\cos\varphi_{10}\omega_{10}\sin\varphi_{30}\omega_{30}-\\-d_3\sin\varphi_{10}\omega_{10}^2\cos\varphi_{30}-d_3\sin\varphi_{10}\cos\varphi_{30}\omega_{30}^2)\\ \\(-d_2\sin\varphi_{20}\omega_{20}^2-d_3\sin\varphi_{30}\omega_{30}^2)\end{bmatrix}\qquad(3.25)$$

4. Discussion

Kinematics of the anthropomorphic systems with velocities and accelerations, may be solved by a basic model 3R, which is a spatial model with matrix calculations (which were presented on this work), or on a 2R planar, simplified model [9].

5. Conclusions

Kinematics of the serial manipulators and robots can be illustrated by a 3R kinematic model, a medium difficulty system, ideal for understanding the phenomenon, but also to specify the basic knowledge necessary for starting calculations for systems simpler and more complex.

The paper presents an original geometrical and kinematic method for the study of geometry and determining positions of a MP-3R structure. It presents shortly the MP-3R direct kinematics with positions, velocities and accelerations.

One presents shortly an original method to solve the robot velocities and accelerations at the 3R-Robots (MP-3R).

If one study (analyze) an anthropomorphic robot with three axes of rotation (which represents the main movements, absolutely necessary), we already have a base system, on which one can then add other movements (secondary, additional). Calculations were arranged and in the matrix form.

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