



## The behaviour of high strength lightweight concretes with the addition of steel fibers

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**Abstract:** The use of high strength lightweight concretes with the addition steel fibers has become more popular in recent days due to their mechanical proprieties. It was possible to observe that structural lightweight concrete has benefits when compared to normal weight concretes due to higher strength-weight ratio and improves tensile strain capability. Moreover, even though it was demonstrated that the shear behavior of lightweight (LWC) are reduced when compared with normal weight (NWC), more studies about shear behavior in LWC beams need to be done. Moreover, the addition of 0.75% of steel fibers by volume, 15% of Fly-ash and 5% of silica fume has demonstrated to be an ideal amount to obtain high-strength concretes with great mechanical properties. Hence, this paper presents a literature review about the behaviour of high strength lightweight concretes with the addition of steel fibers pointing out its mainly mechanical properties.

**Keywords:** Mechanical proprieties; High strength concretes; Lightweight concretes; Fiber addition.

### 1. Introduction

High-performance concrete (HPC) is the one whose “properties and constructability” are higher than normal concrete, and it “must meet a combination of performance requirements” such as “special mixing, placing, and curing practices” (Kosmatka; Kerkhoff; Panarese, 2003). HPCs’ production follows elevate industry standards related to materials quality and the correct proportion of them. The water-cementing materials ratio is an important parameter for HPCs, the lower the water-cement ratio, the greater the performance (Kosmatka; Kerkhoff; Panarese, 2003).

Rakocky and Nowak (2014) state that “structural lightweight concrete provides a more efficient strength-weight ratio in structural elements than normal weight concrete (NWC)”; thus, due to this, the use of lightweight concrete (LWC) for beams and slabs has enhanced. Sajedi and

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Shafiq (2012) emphasize that LWC “is known for its advantage of reducing the self-weight of the structures and areas of sectional members, thus making the construction convenient”.

Concretes with added fibers “are substantially tougher and have greater resistance to cracking and higher impact resistance” because the fiber “has increased the versatility of concrete by reducing its brittleness” and “randomly distributed fibers provide additional strength in all directions” (McCormac; Brown, 2014).

Based on its advantages, the use of steel fibers in structural lightweight concretes has become more popular; hence, this paper presents a literature review about the behaviour of high strength lightweight concretes with the addition of steel fibers.

## **2. High-Performance Concrete**

Kosmatka, Kerkhoff and Panarese (2003) remark that HPCs’ properties will be “developed for particular applications and environments”, and some of those characteristics are: “high strength; high early strength; high modulus of elasticity; high abrasion resistance; high durability and long life in severe environments; low permeability and diffusion; resistance to chemical attack; high resistance to frost and deicer scaling damage; toughness and impact resistance; volume stability; ease of placement; compaction without segregation and inhibition of bacterial and mold growth”.

Kosmatka, Kerkhoff and Panarese (2003) and Patil and Kumbhar (2012) explain that HPC has become very popular, and it has been used in bridges, tall buildings and nuclear power projects.

### **2.1 High-Strength Concrete**

The most common type of HPC is the high-strength concrete (HSC), which sometimes is called HPC (McCormac; Brown, 2014). The National Ready Mixed Concrete Association (2001) and McCormac and Brown (2014) consider that HSCs must have compression strengths greater than 40 MPa (6000 psi).

High-strength concretes can be used for both situations: precast or prestressed members (McCormac; Brown, 2014). According to the National Ready Mixed Concrete Association (2001) and McCormac and Brown (2014), using HSC is interesting and useful for certain reasons. First, a member made with HSC can be put “into service at much earlier age, for example, opening the pavement at 3 days” (National Ready Mixed Concrete Association, 2001). Moreover, HSCs are very useful to build high-rise buildings enabling the construction of smaller and lighter members such as columns and beams. This allows “savings in storage, handling, shipping, and erection costs” (McCormac; Brown, 2014).

In addition to this, HSCs are used “to build the superstructures of long-span -bridges”, to increase “the durability of bridge decks”, to build “dams, grandstand roofs, marine

foundations, parking garages and heavy duty industrial floor” (National Ready Mixed Concrete Association, 2001).

Producing concrete with strengths higher than 40 MPa requires a strict control of the work and the correct selection, proportion and quality of materials that are going to be used (McCormac; Brown, 2014). Producers “must know the factors affecting compressive strength and know how to vary those factors for best results” (Kosmatka; Kerkhoff; Panarese, 2003), and it is required “special care in the selection of the materials to be used” (McCormac; Brown, 2014) “keeping in mind the economic advantages of using locally available materials” (Kosmatka; Kerkhoff; Panarese, 2003).

### **3. Lightweight Concretes**

National Ready Mixed Concrete Association (2003) and Rakoczy and Nowak (2014) state that normal weight concrete (NWC) has density in the range of 2240 to 2400 kg/m<sup>3</sup> (140 to 150 lb/ft<sup>3</sup>), and the lightweight concrete (LWC) has diminished density in the range of 1440 to 1840 kg/m<sup>3</sup> (90 to 115 lb/ft<sup>3</sup>). Wu et al. (2010) remark that “the mechanical properties of LWAC differ significantly from those of normal weight concrete, mainly attributed to the high porosity of LWA, which causes high water absorption rate and smaller modulus of elasticity of concrete thus made”.

Pelisser et al. (2012) point out that “with all the exceptions that may be between different studies and the specific properties of the materials used, a technically and economically vital point is the consumption of cement and the strength obtained, especially considering the application of lightweight concrete or aggregates, and the relation kg/MPa, is a performance index of the concrete with rubber that can be comparable between different studies”.

The National Ready Mixed Concrete Association (2003) presents arguments to emphasize the aspect previously remarked by Pelisser et al. (2012), saying that LWC use for structural purposes aims to reduce the dead load and “reduce the size of columns, footings” and other structural members. In addition to this, LWC can be designed to obtain strength and other mechanical properties and durability performance requirements similar or higher than NWC.

Using LWC presents other important characteristics for buildings. First, as mentioned before, “structural lightweight concrete has advantages of higher strength/weight ratio and better tensile strain capacity”. Additionally, it presents a “lower coefficient of thermal expansion, and superior heat and sound insulation characteristics due to air voids in the lightweight aggregate” (SAJEDI and SHAFIGH, 2012; BARBOSA et al., 2012). Furthermore, “structural lightweight concrete provides a higher fire-rated concrete structure”, and it has beneficial aspects for energy conservation because the lightweight aggregate “provides a source of water for internal curing

of the concrete that provides continued enhancement of concrete strength and durability” (National Ready Mixed Concrete Association, 2003).

LWC has been used for structural purposes for many years (Sajedi and Shafigh, 2012; Rakoczy and Nowak, 2014; Shannag, 2011). Based on this, for its structural utilization, the density is usually more relevant than the strength. LWC decreased density is obtained through using lightweight aggregates (LWA). Bernhardt et al. (2013) advocate, based in various authors, that LWA plays an essential role in LWC strength, such that enhancing and modifying the aggregates’ mechanical properties is necessary.

LWC requires special attention in mixture methods and processes in order to achieve the required mechanical proprieties (National Ready Mixed Concrete Association, 2003; Barbosa et al., 2012; Pelisser et al., 2012; Sajedi and Shafigh, 2012). Barbosa et al. (2012) explain that in case of “expanded aggregates are broken during the mixing process, the hardened concrete strength may be affected” because it intensifies the water absorption coefficient. For instance, Barbosa et al. (2012) state that “the water penetration into broken aggregates causes reduced workability, stiffening the lightweight aggregate concrete”. Moreover, Hossain KMA, cited by Shannag (2011) emphasizes that producing conventional LWA with clay, slate and shale has problems with absorption of huge amounts of mixing water through the porous of those aggregates.

Kilic et al. (2003) report a study made by Alduaij et al., in 1999, in which they obtained a LWC with 22 MPa and dry density of  $1,520 \text{ kg/m}^3$ , at 28 days, by using “lightweight expanded clay and normal weight gravel without the use of natural fine aggregate”. Kilic et al. (2003) conducted a study whose aim was to obtain “more economical and greener high-strength lightweight concrete (HSLC) mixture by the use of mineral admixture fly ash and silica fume together and separately”. Four concrete mixtures with different proportions of fly ash and fume silica were tested, and two control normal weight concrete were made. The mixture M3, which has  $450 \text{ kg/m}^3$  of cement and 10% of silica fume by weight ( $50 \text{ kg/m}^3$ ) and water-cementing (w/c) ratio of 0.55, was the one with hugest compressive strength ( $f_c'$ ) of 38.9 MPa, at 28 days, and 43.8 MPa, at 3 months, and dry density of  $1,820 \text{ kg/m}^3$ . Thus, the authors concluded that it “is possible to produce a lightweight concrete with a 40 MPa compressive strength by the use of silica fume”. Comparing sample M3 with sample CM1, which is made with NWC and same w/c ratio as M3, the M3 sample presented an enhancement of 39.43% in  $f_c'$ , at 28 days.

Shannag (2011) conducted a research “focused on investigating the properties of fresh and hardened concretes containing locally available natural lightweight aggregates, and mineral admixtures” using eleven different types of mixtures with “lightweight aggregates (LWA), cement, silica sand, and admixtures”. Based on Shannag’s study, mixture number 4 was the one with the husgest compressive strength, 43.2 MPa, and hugest modulus of elasticity, 22,477 MPa at 28 days. Mixture number 4 has cement content of  $340 \text{ kg/m}^3$ , the water-cement ratio of 0.63,

slump value of 110 mm and dry density of 1,878 kg/m<sup>3</sup>. Analyzing the results, the author shows that reducing the amount of cement and replacing it with 15% of silica fume increased by 47.44% the compressive strength. Moreover, adding silica fume and fly ash in the most appropriate proportions contributed to “increase in compressive strength and modulus of elasticity compared to individual mixes containing same contents of either silica fume or fly ash”.

Sajedi and Shafigh (2012) report three previous studies in which LWC was obtained by using low density materials. First, in the studies conducted by Norokshchenov and WhitComb (1990), they made a LWC with 70.5 MPa ( 10,225 Kip) and density of 1,860 kg/m<sup>3</sup> by using 520 kg/m<sup>3</sup> of cement, Light Expanded Clay Aggregate (Leca) and “20% of silica fume by percentage of cement weight. The second study analyzed by Sajedi and Safigh (2012) was the one conducted by Rossignolo, Agnesinin and Morais (2003). Through their investigation, they were able to produce a LWC with 53.6 MPa of compressive strength with 1,605 kg/m<sup>3</sup> of dry density “by using Brazilian’s LWC”. The third study was made by Malhotra (1990) who produced LWC of 70 MPa compressive strength, at 365-day, with “dry density 2,000 kg/m<sup>3</sup>”, by using cement of 500 kg/m<sup>3</sup> (type III ASTM), Leca, fly ash, and suspense silica fume.

In their own study, Sajedi and Shafigh (2012) tested the influence of Leca in eleven different types of materials mixtures proportion of LWC. According to the authors, mix number 7 was the one with the highest compressive strength of 67 MPa and highest flexural strength of 9.71 MPa at 28-day, slump value of 100 mm, and dry density of 1,965 kg/m<sup>3</sup>. The cement content was 495 kg/m<sup>3</sup>, the water-cement ratio of 0.29 and 234.2 kg/m<sup>3</sup> of lightweight expanded clay aggregate (Leca). Based on their research, Sajedi and Shafigh (2012) concluded that adding Leca and silica fume contribute to obtaining high-strength lightweight concrete (HSLWC), and limestone plays an important role “in the improvement of mechanical properties of lightweight concretes”. Furthermore, Leca with higher fineness will result in a greater increase in the density and compressive strength of lightweight concretes, and silica fume “fills the voids and improve the compressive strength in HSLWC”.

Therefore, understanding the impact of mixing times, evaluating the most appropriate materials mixture proportions and analyzing the shear and flexural behavior of a member with a LWC is fundamental.

### **3.1 High-Strength Lightweight Concretes**

The structure of high-strength lightweight concretes (HSLWC) “depends on the hydration of cement, crystallization and formation of crystalline splice with cement blider”. The most elevated strength can be obtained through the formation of “the solid object with most density of bonds and strength of single contact” with a very strong connection during hydration (Inozemtce; Korolev, 2014).

Moreno et al. (2014) comment, based on a literature review, that creating a HSLWC is problematic, considering that porous aggregate, water absorption and lightweight influence in an adverse way the mechanical properties and durability of concrete, but HSLWC very often contains added cementing materials that reduce the amount of water used, and they increase its mechanical properties and durability. Inozemtcev (2015) remarks the importance of factors such as “time and rate of mixing, parameters of vibro-compacting and mode of heat-humid treatment (HHT)” on the quality of concrete.

Kılıç et al. (2003) comment on research made by Al-Khaiat and Haque (1998). According to the authors (2003), Al-Khaiat and Haque “worked on the effect of initial curing on early strength and physical properties of lightweight concrete containing 500 kg/m<sup>3</sup> cement and 50 kg/m<sup>3</sup> condensed silica fume”, and they produced a HSLWC with 50 MPa and fresh density of 1,800 kg/m<sup>3</sup>.

Dunbeck (2009) remarked three different studies with HSLWC. The first one was conducted by Meyer and Kahn (2002) that were investigating the benefits of HSLWC. Concretes of 55 MPa (8000 psi), 69 MPa (10,000 psi) and 83 MPa (12,000 psi) “were considered using expanded slate lightweight aggregate”.

The second one was made by Buchberg (2002), who investigated over 75 different mixtures and “developed high-strength lightweight mix designs made with materials available in Georgia”. According to Dunbeck (2009), Buchberg recommended three mixtures of HSLWC of 55 MPa (8000 psi), 69 MPa (10,000 psi) and 83 MPa (12,000 psi). Dunbeck (2009) concluded that silica fume was efficient “in increasing the early strengths of lightweight concrete as well as the late strengths” and the chloride permeability was very low. The mixtures designs found by Buchberg are shown in Table 1.

**Table 1 – Mixtures found by Buchberg - Modified from Dunbeck (2009)**

Material	Units	55 MPa	69 MPa	83 MPa
Type III cement	kg	355	347	336
Flyash	kg	64	66	68
Silica fume	kg	9	22	45
Normal weight fine aggregate	kg	430	433	433
½ inch Stalite aggregate	kg	464	467	467
Water	m <sup>3</sup>	0.122	0.113	0.103
Water/Cementitious Ratio	w/c	0.28	0.26	0.23
Water reducer	mL	1,686	1,715	1,745
Superplasticizer	mL	1,686	1,922	4,111
Air entrainer	mL	278	284	222
Theoretical Wet Unit Weight	kg/ m <sup>3</sup>	1,852	1,861	1,868
Compressive Strength at 56 days	MPa	76.5	77.9	80.1
Elastic Modulus at 56 days	MPa	28,475.3	29,371.7	30,336.9
Chloride Permeability	coulombs category	664 Very Low	300 Very Low	99 Negligible

Dunbeck (2009) ran her own study on the construction of the bridge “I-85 Ramp “B” Bridge over SR-34, Bullsboro Drive, in Cowetta County”, which was built using HSLW concrete girders. Dunbeck (2009) explained that “concrete samples were taken from every batch of concrete used in the construction of the girders”. The materials proportions and the average compressive strength of HSLWC girders are provided in Table 2.

**Table 2 – Mixtures found by Dunbeck**

Material	Units	55 MPa
Cement	kg	1,174.80
Flyash	kg	238.14
Silica fume	kg	158.76
Sand	kg	1,479.62
Lightweight aggregate	kg	1,555.82
Water	m <sup>3</sup>	0.333
Water reducer	mL	3,726.27
High Range Water Reducer	mL	4,672.62
Air Entraining Agent	mL	207.01
Theoretical Wet Unit Weight	kg/ m <sup>3</sup>	1,928.62
Dry Unit Weight	kg/ m <sup>3</sup>	1,860.06
Compressive Strength at 28 days	MPa	66.71
Compressive Strength at 56 days	MPa	70.59
Modulus of Elasticity at 56 days	MPa	25,710.55

Dunbeck (2009) concluded that “silica fume must be closely monitored to ensure that it is mixed well within the batch and the moisture content of the lightweight aggregate should be frequently measured as well to ensure mixture consistency”. Moreover, the average concrete compressive strength was higher than the designed one, but the average modulus of elasticity was 2% lower than the predicted by ACI 363. According to Dunbeck (2009), “previous research showed that the elastic modulus was dependent on the type of lightweight aggregate used even when compressive strengths were the same”.

### 3.2 Shear Behavior in Lightweight Concretes

Shaw (2013) states that preceding studies have demonstrated that “interface surface preparation, reinforcement ratio, concrete strength, and concrete type in terms of unit weight (normalweight, sand-lightweight, or all-lightweight) have significant impacts on the shear transfer strength”.

Emiko et al. (2011) explain that the cracking process in LWC members has certain concerns because this process is related to the cleavage of the aggregate once the aggregate strength is commensurate to the matrix strength, and it result in a “smooth-faced crack” that is not very “effective in transmitting shear stress”.

Sherwood et al., cited by Yang and Ashour (2015), pointed out that increasing the maximum aggregate size from 9.5 to 21 mm (0.37 to 0.83 in), the shear strength of NWC beams had an increment of 24%. Yang et al. (2011) and Yang and Ashour (2015) state that the

aggregate interlock tends to be reduced in LWC beams because inclined cracks emerging through the aggregate particles, create a remarkably brittle fracture surface.

Emiko et al. (2011) made a study whose aim was to investigate “the shear transfer behaviour of reinforced high-strength concrete involved the testing of 14 series of pre-cracked push-off specimens with each series containing three specimens”. Based on this, the shear behavior was compared between NWC and LWC with the authors coming up with certain conclusions. First, it was noted that LWC has shear behaviour identical to NWC, and “test results indicate that the ultimate shear strength increased with an increase in concrete strength and reinforcement parameter”. Second, it was observed that LWC beams had shear transfer strength 20% lower than NWC “for the same amount of reinforcement and the same concrete strength”.

In addition to this, Emiko et al. (2011) showed that the coefficient of friction<sup>1</sup> of a cracked surface is 0.33 for LWC and 0.55 for NWC, and like what happens in NWC, in LWC this coefficient is “independent of concrete strength”. Furthermore, the cohesion of concrete has a lower contribution on shear transfer strength in LWC than in NWC.

Bashe et al. (2006), Emiko et al. (2011) and Yang and Ashour (2015) remark that, even though a huge number of studies have been done about LWC, there is no clear understanding about implications of using LWC and shear behavior in beams made with it.

### **3.3 Flexural Behavior in Lightweight Concretes**

Lim et al. (2006) point out that the principal codes (American – ACI 318, British – BS 8110, European – EC2) have “rules for the design of LWC members”. Lim (2006) and Wu et al. (2011) remark that there are not an expressive amount of literature material about flexural behaviour in LWC, and those that are available are studies conducted in the 1960s, and materials used have evolved significantly. Hence, Wu et al. (2011) and Lim (2006) emphasize how important new research is for improving knowledge, currently formulations, and codes.

Lim et al. (2006) performed research with ten reinforced lightweight aggregate concrete (LWAC) beams with a density of 1,850 kg/m<sup>3</sup>, and one NWC beam, aiming to evaluate their flexural response. Results obtained by the research were compared to the American Code of Practice (ACI 318-2005 and ACI 213-2003). NWC and LWAC beams presented similar data for flexural response: increasing concrete compressive strength, first cracking load increases remarkably, and “the post-cracking stiffness, ultimate strength and ductility” demonstrate minimal increase. In addition to this, they both had similar shear behaviour because “closer spacing of stirrups reduces the amount of drop in the load carrying capacity of a beam at crushing”.

In conclusion, Lim et al. (2006) explain that even though the “American Code of Practice (ACI 318-2005 and ACI 213-2003) for LWAC can predict the cracking and ultimate strength quite accurately, the methods consistently underestimate the service load deflection and

overestimate ductility index in most of the cases”; thus, the calculation recommendations for LWAC beam need to be reviewed.

Wu et al. (2011) conducted a study to investigate “the size effect and the flexural behavior of full size reinforced concrete beams with lightweight aggregate and normal aggregate”. “Flexure beams with various reinforcement ratios (from 0.33% to 1.3%) and 6 size-effect beams with various dimensions were fabricated and tested” with “designed compressive strength of concrete” of 34 MPa (Wu et al., 2011).

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<sup>1</sup> “The friction force is the force exerted by a surface when an object moves across it”. “The coefficient of friction was obtained through the gradient of the shear strength plotted against reinforcement parameter curve”.

Wu et al. (2011) presented the follow conclusions. First, “the reinforced lightweight aggregate concrete (LWAC) beams had a similar load capacity and failure mode as those of normal weight concrete (NWC) beams, but demonstrated larger ultimate deflections and curvature ductility”. Second, “the curvature ductility of both types of concrete beam decreased as the reinforcement ratio increased” (Wu et al., 2011). On other words, LWC beams have the same flexural behavior of NWC beams. Third, “an increase of the beam dimension lead to increases in the load and deflection at the yielding strength and ultimate strength, but reduced the ultimate deflection ratio for both types of concrete beams” (Wu et al., 2011). Fourth, it was identified that the ultimate deflection ratio has different behavior in NWC and LWC when beams size vary because “the ultimate deflection-to-span ratio varied from 1/115 to 1/150 (23% reduction) as the effective depth increased from 300 mm to 600 mm for the NWC beams and from 1/84 to 1/188 (55% reduction) for the LWC beams, respectively” (Wu et al., 2011). Fifth, comparing the tested and calculated mid-span deflection (according to the theory supported by ACI 318,  $\Delta = 0.1ML^2/EI_c$ ) results at cracking and yielding points for NWC and LWAC demonstrated that experimental results at cracking are larger than calculated ones (82% to NWC and 54% to LWAC) (Wu et al., 2011). Contrarily, experimental results at the yielding point were on the average of 10% for both concretes. Hence, these results “indicates that the elastic flexural theory supported by ACI 318 code relatively underestimates the actual deflection for both LWAC and NWC beams” (WU et al., 2011). Both studies, Lim et al. (2006) and Wu et al. (2011), present arguments that propose that the current code should be re-analyzed.

#### **4. Fiber Reinforced Concretes**

According to McCormac and Brown (2014), the interest in fiber reinforced concrete has increased in current days. The fibers used are made from steel, plastics, and glass. Moreover, the

authors (2014) point out that research has shown that adding fibers by the quantity from 1 to 2% by volume have significantly enhanced the normal concrete properties.

Indira and Abraham (2007) note that “conventional concrete loses its tensile resistance after the formation of multiple cracks; however, fiber concrete can sustain a portion of its resistance following cracking to resist more cycles of loading”.

#### **4.1 Steel Fibers**

The most common material used are steel fibers (SF). Jang et al. (2015) remark that the first try of replacing reinforcement steel by steel fibers in RC beams was in 1970.

Ganesan, Indira and Abraham (2007) state that overall, the addition of fibers to concrete increase the “tensile strain in the neighbourhood of fibres”. Furthermore, “the cracking behaviour, ductility, and energy absorption capacity of the composite” will be improved (Ganesan, Indira and Abraham, 2007).

Kang and Kim (2010) remark that steel fibers tend “to improve mechanical properties and structural performance relative to conventionally reinforced concrete (with the same steel volume fraction)”. Furthermore, steel fiber-reinforced concrete (SFRC) members present improvements in shear and flexural behavior. Using SFRC contributes to “the degree and width of cracking”, so there is an improvement of SFRCs members’ behavior related to “the postcracking tensile strength” (Kang; Kim, 2010).

Casanova and Rossi, cited by Jang et al. (2015), conducted a study in 1997, showing that HSC beams without transversal reinforcement with the compressive strength of 90 MPa and 1.25% of steel fibers “could obtain equivalent performances of HSC beams with 1.1% conventional transverse reinforcements”.

Kwak et al. (2002) tested twelve beams: nine high-strength steel fiber-reinforced concretes (HS-SFRC) (65 MPa), with volume fractions ( $V_f$ ) of 0, 0.5 and 0.75% and shear span-depth ratio ( $a/d$ ) equals to 2, 3 and 4; and three beams (31 MPa) with 0.5% of steel fiber by volume fraction and  $a/d$  of 2, 3 and 4. “The longitudinal bars were hooked upwards behind the supports and no stirrups were included within the shear span” (KWAK et al., 2002).

Kwak et al. (2002) observed that steel fibres remarkably reduced the crack width and size, increased deformation capacity, and contributed to creating a ductile mode instead of a brittle one. Moreover, the failure mode varied according to the  $V_f$  and  $a/d$  variations. For  $a/d$  equal to 2, two different types of failure occurred: for  $V_f = 0\%$ , the beam failed in shear; for  $V_f = 0.5\%$  and  $V_f = 0.75\%$ , it failed in shear-flexure. For  $a/d$  equal to 3 and 4 and  $V_f = 0\%$ , the beam failed in shear; for  $V_f = 0.5\%$  and  $V_f = 0.75\%$ , it failed in flexure. Failing in flexure means that “the applied load at failure is not equal to the shear strength” (Kwak et al., 2002). Regardless, beams with the smallest  $a/d$  ratio ( $a/d = 2$ ) had a huge increment in their shear strength from 69 to 80%. On the other hand, beams with larger  $a/d$  ratios had smaller shear strength improvement (22 to 38%).

Jang et al. (2015) reported another study, which was conducted by Cucchiara et al., in 2004, in which they tested hooked-end SFRC beams “with different amount of shear reinforcement, shear-to-span ratio, and fibre volume fraction”. The results demonstrated that adding “hooked-end steel fibres in the shear-dominant RC beams can transform the brittle behaviour characterized by shear failure into a ductile one by flexural failure” (Jang et al., 2015). Jang et al. (2015) reported other research done by different people, and all of them observed improvements in the cracks development.

Jang et al. (2015) conducted a study to investigate “the influence of steel fibre contents on the mechanical properties of HPC” of beams with compressive strength of 60 MPa and 100 MPa. Steel fibers tested quantities were 0, 0.5, 1.0 and 1.5% by volume, and they presented the following conclusions. First, there were no problems in mixing and casting up hooked-end steel fibres with a volume fraction of 1.5%. By increasing the amount of SF, the air content (AC) increases and the slump values decrease, which makes it less workable. Second, using 1.5% of steel fibers by volume led to an increment of 42.3% in the modulus of rupture of 60 MPa concrete beam, and 30.0% in the 100 MPa. Third, the “replacement of minimum shear reinforcement with deformed steel fibres” is 1.2% by volume to 60 MPa and 1.5% by volume to 100 MPa. Fourth, “aspect ratio (length over diameter ratio of hooked-end fibers) of 2.0 is effective to inhibit the crack development and decrease the increasing rate of shear span” Jang et al., 2015). Fifth, a high-strength steel fiber-reinforced concrete (HS-SFRC) beam with 1.5% of SF in a 60 MPa concrete (with no transverse reinforcements) demonstrated similar shear resistance of conventional beams with transverse reinforcements full confined.

Another field of application of SF can be found in LWC members. Kang and Kim (2010), Kang et al. (2011) and Campione (2014) point out that even though ACI 318-08 establishes the minimum of 0.75% of steel fiber fraction by volume, there are not sufficient studies that explain mechanical properties and shear and flexural behavior of steel fiber-reinforced lightweight concrete (SFRLWC).

Kang and Kim (2010) and Kang et al. (2011) remark a study made by Swamy et al., in 1993, who tested I-section beams. Their test results indicated that SFRLWC with “a steel fiber volume fraction of 1% showed significantly greater shear strength (by 60% to 210%) than equivalent beams without steel fibers”.

Kang et al. (2011) performed a study with twelve concrete beams (six SFRLWC, three SFRC, and three LWC) “without stirrups were simply supported and loaded with two equal concentrated loads using a spreader steel beam”. Three volumes of SF ( $V_f$ ) were tested, 0, 0.5 and 0.75%, and three different a/d ratios, 2, 3 and 4. Unit weights of normal and lightweight concrete tested were 2,194 and 1,800 kg/m<sup>3</sup>, the water-cement ratio was 0.33, for all beams, and the compressive strength of concrete varied from 39.6 to 57.2 MPa.

Based on tests results, Kang et al. (2011) presented certain conclusions. First, the compressive strength of SFRLWC increased with the increasing of volume of added fibers (by 13% for  $V_f = 0.5\%$  and 20% for  $V_f = 0.75\%$ ). Moreover, the tensile strength of SFRLWC was also increased by 40% for  $V_f = 0.5\%$ , and approximately 70% to 100% for a  $V_f$  of 0.75%. Second, SFRC material properties demonstrated huger values than SFRLWC, “compressive strength ( $f_c$ ) by 28%, splitting tensile strength ( $f_{sp}$ ) by 33%, modulus of rupture ( $f_r$ ) by 14%, and modulus of elasticity ( $E_c$ ) by 20% on average” (KANG et al., 2011). Thus, SFRC had a larger shear capacity than SFRLWC. Third, by adding steel fibers (comparing with no fibres), the resistance to structural derange, ductility and shear capacity enhanced, and increasing the volume of fibers led to “a change in the failure mode from brittle to ductile”. Fourth, Kang et al. (2011) advocates that “the ACI 318-08 minimum requirement of 0.75% appears to also be reasonable for steel fiber-reinforced lightweight concrete beams” because SFRLWC with  $V_f = 0.75\%$  or  $V_f = 0.5\%$  and  $a/d = 2$  to 3 “performed well without any signs of brittle failure”. Fifth, as the  $a/d$  ratio increased, the shear stress at diagonal crack and peak decreased. It is important to note that the current equations of SFRLWC do not consider  $a/d$  influence, so they should be incorporated.

#### 4.2 Mechanical Properties of Steel Fiber-Reinforced Lightweight Concrete

In this section, results of five different studies, which used fibers in the concrete, will be discussed.

The first study was conducted by Gao, Sun and Morino (1997), who tested five different mixtures of high strength concrete, reinforced with steel fibers. Rectangular fibres with lengths of 20, 25 and 30 mm, aspect ratios ( $l/d$ ) of 46, 58 and 70, respectively and the volume of fibres ( $V_f$ ) used were 0, 0.6, 1.0, 1.5 and 2.0%, with water-cementing ratio of 0.28. According to Gao, Sun and Morino (1997), “the compressive strength ( $f_c$ ) and the splitting tensile strength ( $f_{sp}$ ) were measured on 100 x 100 x 100 mm cubes, the flexural strength ( $f_r$ ) was tested on 100 x 100 x 400 mm specimens with four-point flexural loading”. Moreover, “the modulus of elasticity ( $E_c$ ) was calculated, based on the stress corresponding to 40% of ultimate strength and the longitudinal strain produced by this stress” (Gao; Sun; Morino, 1997).

According to Gao, Sun and Morino (1997), results are the following. First, the compressive strength ( $f_c$ ) demonstrated a very small increase with increasing of  $V_f$ , from 70.2 to 85.4 MPa, and it can be explained, according to authors (1997), due to “the ultimate strength of concrete be controlled by the strength of lightweight aggregates”. Second, the splitting tensile strength ( $f_{sp}$ ) increased significantly, from 4.95 to 8.88 MPa (19-78%), “depending on the various fibre volume and aspect ratio”, but to have an effective impact in  $f_{sp}$ , the volume of fibres must be over 1% by volume (Gao; Sun; Morino, 1997). In addition to this, Gao, Sun and Morino (1997) remark that “the splitting strength increases linearly with the addition of fibres and is linear functions of  $V_f$  and  $l/d$ . Third, the flexural strength ( $f_r$ ) strongly enhanced form 6.2

to 11.8 MPa (9.6 to 90%) according to  $V_f$  and aspect ratio. Gao, Sun and Morino (1997) explain that added fibres will carry the load, and they will bond the cracks, thus “the deflection corresponded to ultimate load increases with the increase of fiber volume and aspect ratio”. Fourth, the modulus of elasticity ( $E_c$ ) is significantly affected by the lightweight aggregates used, which are porous, so  $E_c$  tends to be lower using lightweight aggregates than normal ones. On the other hand, steel fibres have elevated  $E_c$ , which contributes to enhancing the  $E_c$  of the concrete mixture, so the  $E_c$  varied from 23.1 to 27.9 GPa depending on  $V_f$  and aspect ratios (Gao; Sun; Morino, 1997).

Thomas and Ramaswamy (2007) made an experimental program in which they tested the influence of steel fibers in three different types of concrete: normal-strength concrete (35 MPa, w/c = 0.48), moderately high-strength concrete (65 MPa, w/c = 0.35) and high-strength concrete (85 MPa, w/c = 0.28). Steel fibres used had 30 mm of length and aspect ratio of 55 and  $V_f$  of 0, 0.5, 1.0 and 1.5%. The compressive strength was tested in two different specimens: cube (150 x 150 x 150) and cylinder (150 $\phi$  x 300); the splitting tensile strength with a cylinder of 150 $\phi$  x 300; the modulus of rupture was tested using a prism with 100 x 100 x 100; and the modulus of elasticity using a cylinder of 150 $\phi$  x 300.

Thomas and Ramaswamy (2007) remarked the following conclusions. First, the increase of compressive strength was not symbolic. Using the cube compressive strength, from 0 to 1.5% by volume of fibres, it was observed an increment of “3.65% in normal-strength concrete, 2.65% in moderately high-strength concrete, and 2.59% in high-strength concrete”, and the cylinder compressive strength was “8.33% in normal-strength concrete, 6.10% in moderately high-strength concrete, and 4.60% in high-strength concrete” (Thomas and Ramaswamy, 2007). Second, split tensile strength enhanced largely, by 38.2% in normal-strength concrete, 41.2% in moderately high-strength concrete, and 38.5% in high-strength concrete. Third, the modulus of rupture had a considerable increment by using fibres: 46.2% in normal-strength concrete, 38.8% in moderately high-strength concrete, and 40.0% in high-strength concrete. On other words, those results mean a remarkable enhancement in post-cracking response “with fibres dosages across the different concrete grades” (Thomas and Ramaswamy, 2007). Fourth, the modulus of elasticity was not somewhat affected by the addition of fibres once it increment was 8.3% in normal-strength concrete, 9.2% in moderately high-strength concrete, and 8.2% in high-strength concrete.

Wang and Wang (2013) report a study in which “five groups of SFLWC specimens with different steel fiber volumes including 0.0%, 0.5%, 1.0%, 1.5% and 2.0% were tested to investigate the effect of steel fiber content on the static mechanical properties and the impact resistance of lightweight aggregate concrete”. The compressive strength, at 28 days, of LWC was 60.4 MPa and the water-cement ratio of 0.42 (Wang; Wang, 2013). The compressive strength and the splitting tensile strength were tested using specimens of 150 x 150 x 150 mm,

and the flexural strength was tested “on 150 x 150 x 550 mm specimens with four-point flexural loading” (Wang; Wang, 2013).

Wang and Wang (2013) come up with certain conclusions. First, “test results show the compressive strength varied from 60.4 MPa to 74.8 MPa (23.8%), corresponding to the age of 28 days for the various fiber volume fractions” (Wang; Wang, 2013). Like it was mentioned by Gao, Sun and Morino (1997), in LWC, lightweight coarse aggregates control the ultimate strength of concrete. However, “the incorporation of steel fiber into matrix serves to increase the ultimate compressive strength by the resultant arresting growth of cracks based on the bond of steel fiber and cement paste” (Wang; Wang, 2013). Second, the splitting tensile strength demonstrated a largely enhancement, from 3.99 to 7.6 MPa (92.5%). Third, the flexural strength also increased due to “the influence of fibre arresting cracking” (Wang; Wang, 2013). In addition to this, Wang and Wang (2013) observe the way that LWC and SFLWC discs failed: “the SFLWC discs failed largely by the two-piece break, whereas the LWC discs failed mostly by the three-piece break shown” (Wang; Wang, 2013). Moreover, in SFLWC breaking pieces were connected with fibers, and in LWC (with no fibres), the broken parts were separated (Wang; Wang, 2013).

Iqbal et al. (2015) made a study to investigate “mechanical properties of steel fiber reinforced high-strength lightweight self-compacting concrete (SHLSCC)”. Compressive strength, splitting tensile strength and modulus of elasticity were tested using cylinder specimens of 100 mm (diameter) x 200 mm (height), at the age of 28 days. Flexural tests were made using small prisms of 80 x 80 x 400 mm, also at the age of 28 days. Steel fibers with length of 13 mm and aspect ratio of 65 were used, and by the volume fraction of 0, 0.5, 0.75, 1.0 and 1.25%. Water-cementing ratio was 0.46 for  $V_f$  of 0, 0.5, 0.75 and 1.0%, and it was 0.48 for  $V_f$  1.25%.

Based on their study, Iqbal et al. (2015) present following results. First, the compressive strength demonstrated a small reduction (12%) when the  $f_c$  (at 28 days) of concrete without fibers (67.80 MPa) was compared with  $f_c$  (at 28 days) of concrete with 1.25 % of fibres volume fraction (59.74 MPa). According to Iqbal et al. (2015), this reduction is “due to the increase of air content” with increasing of steel fiber content. Second, the splitting tensile strength increased with increasing of steel fibers volume, from 4.1 to 5.64 MPa (37%). Third, the flexural strength also increased from 3.7 to 7.62 MPa, and “the first crack load increases by around 32% while there is an increase of around 110% in peak load, once the fiber content is increased from 0% to 1.25%, once the fibers start bridging the cracks increasing the ultimate load” (Iqbal et al., 2015). Fourth, even though the modulus of elasticity reduces if compare the concrete without fibers with those that have fibers, Iqbal et al. (2015) state that “the modulus of elasticity remains unaffected by the addition of steel fibers”.

As it was mentioned in the previous section, Jang et al. (2015) conducted a study to investigate “the influence of steel fibre contents on the mechanical properties of HPC” of beams with the compressive strength of 60 MPa and 100 MPa. Steel fibers tested quantities were 0, 0.5, 1.0 and 1.5% by volume, and the average length of the fibers are 30 mm, and aspect ratio of 60. Prismatic specimens of 100 x 100 x 400 mm were used for flexural strength test, and cylindrical specimens of 100 x 200 mm were used for compressive strength test. Furthermore, “to investigate the feasibility of replacing stirrup and additional transverse reinforcement with hooked-end steel fibres for the shear-dominant short coupling beams, two specimens were designed, constructed, and tested up to failure” (Jang et al., 2015). Each beam is 1300 mm long, and they have the cross section of 200 x 300 mm.

According to the study, Jang et al. (2015) present following conclusions. First, increasing the volume of added steel fibres, the compressive strength reduced in 60 MPa and 100 MPa samples, if compared those without and with steel fibres. Jang et al. (2015) remark that “the presence of hooked-end steel fibres had little effect on the compressive strength of HP-SFRC with specific compressive strength of 60 and 100MPa”. Second, the modulus of rupture had significant increment with the increasing of steel fibers by volume. Jang et al. (2015) observe that “the addition of 1.5% steel fibres to the 60MPa and 100MPa HPC caused a maximum increase of 42.3% and 30.0% compared with the modulus of rupture of HPC without steel fibres, respectively”. Third, using 1.5% of steel fibers by volume, in the 60 MPa concrete is sufficient to create a ductility behavior in the tested beams.

## 5. Conclusions

This literature review paper presented information regarding the behaviour of high strength lightweight concretes with the addition of steel fibers, and their mainly mechanical properties were demonstrated through different and well conducted researches. It was possible to observe that structural lightweight concrete has benefits when compared to normal weight concretes due to higher strengthens-weight ratio and improves tensile strain capability. Moreover, even though it was demonstrated that the shear behavior of LWC are reduced when compared with NWC, more studies about shear behavior in LWC beams need to be done. Regarding the flexural behavior, LWC has bigger ultimate deflections, and elastic flexural theory proposed by ACI 318 code somewhat underestimates the actual deflection for LWC.

Another aspect investigated in this paper was the addition of steel fibers. Studies analyzed in this paper have shown that steel fibers increase shear and flexural behaviour. Adding 0.75% of fibers by volume demonstrated a huge improvement in shear and tensile strength. Furthermore, steel fibers extraordinarily reduced the crack width and size, enhanced the deformation capacity, and contributed to creating a ductile mode instead of a brittle one.

Although all studies presented here have publicised remarkable improvements in shear and flexural behaviours of High-Strength Fibre-Reinforced Lightweight Concretes, it is fundamental that new research projects be conducted. These prospective studies will contribute to a better understanding, and they will contribute to change current codes and create innovative ones.

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