GEAR DESIGN

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Abstract: *The paper presents an original method to determine the efficiency of the gear, the forces of the gearing, the velocities and the powers. The originality of this method relies on the eliminated friction modulus. The first chapters are analyzing the influence of a few parameters concerning gear efficiency. These parameters are:* z_1 *- the number of teeth for the primary wheel of gear;* z_2 *- the number of teeth of the secondary wheel of gear; alpha*$_0$ *- the normal pressure angle on the divided circle; beta - the inclination angle. With the relations presented in this paper, it can synthesize the gear's mechanisms. Today, the gears are present everywhere, in the mechanical's world (In vehicle's industries, in electronics and electro-technique equipments, in energetically industries, etc.). Optimizing this mechanism (the gears mechanism), we can improve the functionality of the transmissions with gears. At the gear mechanisms an important problem is the interference of the teeth. To avoid the interference between teeth, we must know the minimum number of teeth of the driving wheel, in function of the pressure angle (normal on the pitch circle, alpha0), in function of the tooth inclination angle (beta), and in function transmission ratio (i). The last chapter presents an original method to make the geometric synthesis of the gear, having in view the minimum number of teeth of the driving wheel. The classical methods use many different relations to determine the minimum number of teeth of the driving wheel. By this paper we want to give a unitary method to determine the minimum number of teeth of the driving wheel 1, to avoid the interference between the teeth of the two wheels (of the gear).*

Keywords: *gear, gear dynamics, gear efficiency, avoid the interference, forces of gear*

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1. INTRODUCTION

Gears, broke today in all fields. They have the advantage of working with very high efficiency. Additionally gears can transmit large loads. Regardless of their size, gear must be synthesized carefully considering the specific conditions. This paper tries to present the main conditions that must be met for correct synthesis of a gear [1-12].

In the second paragraph the authors present an original method for calculating the efficiency of the gear, the forces of the gearing, the velocities and the powers.

The originality consists in the way of determination of the gear efficiency because it hasn't used the friction forces of couple (this new way eliminates the classical method).

It eliminates the necessity of determining the friction coefficients by different experimental methods as well.

The efficiency determinates by the new method is the same like the classical efficiency, namely the mechanical efficiency of the gear.

Precisely one determines the dynamics efficiency, but at the gears transmissions, the dynamics efficiency is the same like the mechanical efficiency; this is a greater advantage of the gears transmissions [1-12].

It shows the forces, speeds, load and power, which acting in couple.

In the third paragraph one presents shortly an original method to obtain the efficiency of the geared transmissions in function of the contact ratio. With the presented relations it can make the dynamic synthesis of the geared transmissions having in view increasing the efficiency of gearing mechanisms in work [4].

Optimizing this mechanism (the gears mechanism), we can improve the functionality of the transmissions with gears.

At the gear mechanisms an important problem is the interference of the teeth. To avoid the interference between teeth, we must know the minimum number of teeth of the driving wheel, in function of the pressure angle (normal on the pitch circle, $alpha_0$), in function of the tooth inclination angle (beta), and in function transmission ratio (i).

The last chapter presents an original method to make the geometric synthesis of the gear, having in view the minimum number of teeth of the driving wheel. The classical methods use many different relations to determine the minimum number of teeth of the driving wheel. By this paper we want to give a unitary method to determine the minimum number of teeth of the driving wheel 1, to avoid the interference between the teeth of the two wheels (of the gear).

2. GEAR DESIGN

2.1 Determining the momentary dynamic (mechanical) efficiency, the forces of the gearing, and the velocities

The calculating relations [2, 3], are the next (2.1-2.21), (see the Fig. 1):

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$$\begin{cases} F_\tau = F_m \cdot \cos\alpha_1 \\ F_\psi = F_m \cdot \sin\alpha_1 \\ v_2 = v_1 \cdot \cos\alpha_1 \\ v_{12} = v_1 \cdot \sin\alpha_1 \\ \overline{F_m} = \overline{F_\tau} + \overline{F_\psi} \\ \overline{v_1} = \overline{v_2} + \overline{v_{12}} \end{cases} \quad (2.1)$$

with: F_m - the motive force (the driving force);

$F\tau$ - the transmitted force (the useful force);

$F\psi$ - the slide force (the lost force);

v_1 - the velocity of element 1, or the speed of wheel 1 (the driving wheel);

v_2 - the velocity of element 2, or the speed of wheel 2 (the driven wheel);

v_{12} - the relative speed of the wheel 1 in relation with the wheel 2 (this is a sliding speed).

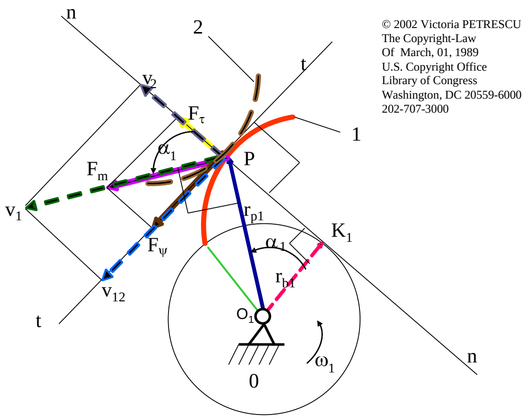


Fig. 1. *The forces and the velocities of the gearing*

The consumed power (in this case the driving power):

$$P_c \equiv P_m = F_m \cdot v_1 \quad (2.2)$$

The useful power (the transmitted power from the profile 1 to the profile 2) will be written:

$$P_u \equiv P_\tau = F_\tau \cdot v_2 = F_m \cdot v_1 \cdot \cos^2\alpha_1 \quad (2.3)$$

The lost power will be written:

$$P_\psi = F_\psi \cdot v_{12} = F_m \cdot v_1 \cdot \sin^2\alpha_1 \quad (2.4)$$

The momentary efficiency of couple will be calculated directly with the next relation:

$$\begin{cases} \eta_i = \dfrac{P_u}{P_c} \equiv \dfrac{P_\tau}{P_m} = \dfrac{F_m \cdot v_1 \cdot \cos^2\alpha_1}{F_m \cdot v_1} \\ \eta_i = \cos^2\alpha_1 \end{cases} \quad (2.5)$$

The momentary losing coefficient [1], will be written:

$$\begin{cases} \psi_i = \dfrac{P_\psi}{P_m} = \dfrac{F_m \cdot v_1 \cdot \sin^2\alpha_1}{F_m \cdot v_1} = \sin^2\alpha_1 \\ \eta_i + \psi_i = \cos^2\alpha_1 + \sin^2\alpha_1 = 1 \end{cases} \quad (2.6)$$

It can easily see that the sum of the momentary efficiency and the momentary losing coefficient is 1:

Now, one can determine the geometrical elements of gear. These elements will be used in determining the couple efficiency, η.

2.2 The geometrical elements of the gear

We can determine the next geometrical elements of the external gear, [2-3], (for the right teeth, β=0):

The radius of the basic circle of wheel 1 (of the driving wheel), (2.7):

$$r_{b1} = \frac{1}{2} \cdot m \cdot z_1 \cdot \cos\alpha_0 \quad (2.7)$$

The radius of the outside circle of wheel 1 (2.8):

$$r_{a1} = \frac{1}{2} \cdot (m \cdot z_1 + 2 \cdot m) = \frac{m}{2} \cdot (z_1 + 2) \quad (2.8)$$

It determines now the maximum pressure angle of the gear (2.9):

$$\cos \alpha_{1M} = \frac{r_{b1}}{r_{a1}} = \frac{\frac{1}{2} \cdot m \cdot z_1 \cdot \cos \alpha_0}{\frac{1}{2} \cdot m \cdot (z_1 + 2)} = \frac{z_1 \cdot \cos \alpha_0}{z_1 + 2} \quad (2.9)$$

And now one determines the same parameters for the wheel 2, the radius of basic circle (2.10) and the radius of the outside circle (2.11) for the wheel 2:

$$r_{b2} = \frac{1}{2} \cdot m \cdot z_2 \cdot \cos \alpha_0 \quad (2.10)$$

$$r_{a2} = \frac{m}{2} \cdot (z_2 + 2) \quad (2.11)$$

Now it can determine the minimum pressure angle of the external gear (2.12, 2.13):

$$\begin{cases} tg\alpha_{1m} = \frac{N}{r_{b1}} \\ N = (r_{b1} + r_{b2}) \cdot tg\alpha_0 - \sqrt{r_{a2}^2 - r_{b2}^2} = \\ = \frac{1}{2} \cdot m \cdot (z_1 + z_2) \cdot \sin \alpha_0 - \\ - \frac{m}{2} \cdot \sqrt{(z_2 + 2)^2 - z_2^2 \cdot \cos^2 \alpha_0} = \\ = \frac{m}{2} \cdot [(z_1 + z_2) \cdot \sin \alpha_0 - \\ - \sqrt{z_2^2 \cdot \sin^2 \alpha_0 + 4 \cdot z_2 + 4}] \end{cases} \quad (2.12)$$

$$tg\alpha_{1m} = [(z_1 + z_2) \cdot \sin \alpha_0 - \sqrt{z_2^2 \cdot \sin^2 \alpha_0 + 4 \cdot z_2 + 4}]/(z_1 \cdot \cos \alpha_0) \quad (2.13)$$

Now we can determine, for the external gear, the minimum (2.13) and the maximum (2.9) pressure angle for the right teeth. For the external gear with bended teeth (β≠0) it uses the relations (2.14, 2.15 and 2.16):

$$tg\alpha_t = \frac{tg\alpha_0}{\cos \beta} \quad (2.14)$$

$$tg\alpha_{1m} = [(z_1 + z_2) \cdot \frac{\sin \alpha_t}{\cos \beta} - \sqrt{z_2^2 \cdot \frac{\sin^2 \alpha_t}{\cos^2 \beta} + 4 \cdot \frac{z_2}{\cos \beta} + 4}] \cdot \frac{\cos \beta}{z_1 \cdot \cos \alpha_t} \quad (2.15)$$

$$\cos \alpha_{1M} = \frac{\frac{z_1 \cdot \cos \alpha_t}{\cos \beta}}{\frac{z_1}{\cos \beta} + 2} \quad (2.16)$$

For the internal gear with bended teeth (β≠0) it uses the relations (2.14 with 2.17, 2.18-A, or with 2.19, 2.20-B):

A. <u>When the driving wheel 1, has external teeth:</u>

$$tg\alpha_{1m} = [(z_1 - z_2) \cdot \frac{\sin \alpha_t}{\cos \beta} + \sqrt{z_2^2 \cdot \frac{\sin^2 \alpha_t}{\cos^2 \beta} - 4 \cdot \frac{z_2}{\cos \beta} + 4}] \cdot \frac{\cos \beta}{z_1 \cdot \cos \alpha_t} \quad (2.17)$$

$$\cos\alpha_{1M} = \frac{\dfrac{z_1 \cdot \cos\alpha_t}{\cos\beta}}{\dfrac{z_1}{\cos\beta} + 2} \qquad (2.18)$$

B. <u>When the driving wheel 1, have internal teeth:</u>

$$tg\alpha_{1M} = [(z_1 - z_2) \cdot \frac{\sin\alpha_t}{\cos\beta} + \\ + \sqrt{z_2^2 \cdot \frac{\sin^2\alpha_t}{\cos^2\beta} + 4 \cdot \frac{z_2}{\cos\beta} + 4}] \cdot \frac{\cos\beta}{z_1 \cdot \cos\alpha_t} \qquad (2.19)$$

$$\cos\alpha_{1m} = \frac{\dfrac{z_1 \cdot \cos\alpha_t}{\cos\beta}}{\dfrac{z_1}{\cos\beta} - 2} \qquad (2.20)$$

2.3 Determining the efficiency of the gear

The efficiency of the gear will be calculated through the integration of momentary efficiency on all sections of gearing movement, namely from the minimum pressure angle to the maximum pressure angle, the relation (2.21), [2, 3].

$$\eta = \frac{1}{\Delta\alpha} \cdot \int_{\alpha_m}^{\alpha_M} \eta_i \cdot d\alpha = \frac{1}{\Delta\alpha} \int_{\alpha_m}^{\alpha_M} \cos^2\alpha \cdot d\alpha = \\ = \frac{1}{2 \cdot \Delta\alpha} \cdot [\frac{1}{2} \cdot \sin(2 \cdot \alpha) + \alpha]_{\alpha_m}^{\alpha_M} = \\ = \frac{1}{2 \cdot \Delta\alpha} [\frac{\sin(2\alpha_M) - \sin(2\alpha_m)}{2} + \Delta\alpha] = \\ = \frac{\sin(2 \cdot \alpha_M) - \sin(2 \cdot \alpha_m)}{4 \cdot (\alpha_M - \alpha_m)} + 0.5 \qquad (2.21)$$

2.4 The calculated efficiency of the gear

We shall see now, four tables with the calculated efficiency depending on the input parameters, and once we proceed with the results we will draw some conclusions.

The input parameters are: z_1=the number of teeth for the driving wheel 1; z_2=the number of teeth for the driven wheel 2, or the ratio of transmission, i ($i_{12}=-z_2/z_1$); α_0=the pressure angle normal on the divided circle; β=the bend angle.

Table 1. *Determining the efficiency of the gear's right teeth for $i_{12effective}$= - 4*

i_{12}=-4	right teeth				
$z_1=8$	$z_2=32$		$z_1=30$	$z_2=120$	
$\alpha_0=20^0$ →	$\alpha_0=29^0$	$\alpha_0=35^0$	$\alpha_0=15^0$	$\alpha_0=20^0$	$\alpha_0=30^0$
$\alpha_m=-16.2^0$	$\alpha_m=0.715^0$	$\alpha_m=11.13^0$	$\alpha_m=1.506^0$	$\alpha_m=9.5367^0$	$\alpha_m=23.125^0$
$\alpha_M=41.257^0$	$\alpha_M=45.597^0$	$\alpha_M=49.056^0$	$\alpha_M=25.101^0$	$\alpha_M=28.241^0$	$\alpha_M=35.718^0$
	η=0.8111	η=0.7308	η=0.9345	η=0.8882	η=0.7566
$z_1=10$	$z_2=40$		$z_1=90$	$z_2=360$	
$\alpha_0=20^0$ →	$\alpha_0=26^0$	$\alpha_0=30^0$	$\alpha_0=8^0$ →	$\alpha_0=9^0$	$\alpha_0=20^0$
$\alpha_m=-9.89^0$	$\alpha_m=1.307^0$	$\alpha_m=8.221^0$	$\alpha_m=-0.163^0$	$\alpha_m=1.5838^0$	$\alpha_m=16.499^0$
$\alpha_M=38.456^0$	$\alpha_M=41.496^0$	$\alpha_M=43.806^0$	$\alpha_M=14.363^0$	$\alpha_M=14.935^0$	$\alpha_M=23.181^0$
	η=0.8375	η=0.7882		η=0.9750	η=0.8839
$z_1=18$	$z_2=72$				
$\alpha_0=19^0$	$\alpha_0=20^0$	$\alpha_0=30^0$			
$\alpha_m=0.9860^0$	$\alpha_m=2.7358^0$	$\alpha_m=18.283^0$			
$\alpha_M=31.683^0$	$\alpha_M=32.250^0$	$\alpha_M=38.792^0$			
η=0.90105	η=0.8918	η=0.7660			

We begin with the right teeth (the toothed gear), with i=-4, once for z_1 we shall take successively different values, rising from 8 teeth. It can see that for 8 teeth of the driving wheel the standard pressure angle, $\alpha_0=20^0$, is so small to be used (it obtains a minimum pressure angle, α_m, negative and this fact is not admitted!).

In the second table we shall diminish (in module) the value for the ratio of transmission, i, from 4 to 2. It will see how for a lower value of the number of teeth of the wheel 1, the standard pressure angle ($\alpha_0=20^0$) is too small and it will be necessary to increase it to a minimum value. For example, if $z_1=8$, the necessary minimum value is $\alpha_0=29^0$ for i=-4 (see the table 1) and $\alpha_0=28^0$ for i=-2 (see the table 2). If $z_1=10$, the necessary minimum pressure angle is $\alpha_0=26^0$ for i=-4 (see the table 1) and $\alpha_0=25^0$ for i=-2 (see the table 2).

Table 2. *Determining the efficiency of the gear's right teeth for $i_{12effective}$= - 2*

$i_{12}=-2$	right teeth				
$z_1=8$	$z_2=16$		$z_1=18$	$z_2=36$	
$\alpha_0=20^\circ$?	$\alpha_0=28^\circ$	$\alpha_0=35^\circ$	$\alpha_0=18^\circ$	$\alpha_0=20^\circ$	$\alpha_0=30^\circ$
$\alpha_m=-12.65^\circ$?	$\alpha_m=0.9149^\circ$	$\alpha_m=12.2835^\circ$	$\alpha_m=0.6756^\circ$	$\alpha_m=3.9233^\circ$	$\alpha_m=18.6935^\circ$
$\alpha_M=41.2574^\circ$	$\alpha_M=45.0606^\circ$	$\alpha_M=49.0559^\circ$	$\alpha_M=31.1351^\circ$	$\alpha_M=32.2505^\circ$	$\alpha_M=38.7922^\circ$
	$\eta=0.8141$	$\eta=0.7236$	$\eta=0.9052$	$\eta=0.8874$	$\eta=0.7633$
$z_1=10$	$z_2=20$		$z_1=90$	$z_2=180$	
$\alpha_0=20^\circ$?	$\alpha_0=25^\circ$	$\alpha_0=30^\circ$	$\alpha_0=8^\circ$	$\alpha_0=20^\circ$	$\alpha_0=30^\circ$
$\alpha_m=-7.13^\circ$?	$\alpha_m=1.3330^\circ$	$\alpha_m=9.4106^\circ$	$\alpha_m=0.5227^\circ$	$\alpha_m=16.5667^\circ$	$\alpha_m=27.7823^\circ$
$\alpha_M=38.4568^\circ$	$\alpha_M=40.9522^\circ$	$\alpha_M=43.8060^\circ$	$\alpha_M=14.3637^\circ$	$\alpha_M=23.1812^\circ$	$\alpha_M=32.0917^\circ$
	$\eta=0.8411$	$\eta=0.7817$	$\eta=0.9785$	$\eta=0.8836$	$\eta=0.7507$

When the number of teeth of the wheel 1 increases, it can decrease the normal pressure angle, α_0. One shall see that for $z_1=90$ it can take less for the normal pressure angle (for the pressure angle of reference), $\alpha_0=8^0$. In the table 3 it increases the module of i, value (for the ratio of transmission), from 2 to 6.

Table 3. *Determining the efficiency of the gear's right teeth for $i_{12effective}$= - 6*

$i_{12}=-6$	right teeth				
$z_1=8$	$z_2=48$		$z_1=18$	$z_2=108$	
$\alpha_0=20^\circ$?	$\alpha_0=30^\circ$	$\alpha_0=35^\circ$	$\alpha_0=19^\circ$	$\alpha_0=20^\circ$	$\alpha_0=30^\circ$
$\alpha_m=-17.86^\circ$?	$\alpha_m=1.7784^\circ$	$\alpha_m=10.660^\circ$	$\alpha_m=0.4294^\circ$	$\alpha_m=2.2449^\circ$	$\alpha_m=18.1280^\circ$
$\alpha_M=41.2574^\circ$	$\alpha_M=46.1462^\circ$	$\alpha_M=49.0559^\circ$	$\alpha_M=31.6830^\circ$	$\alpha_M=32.2505^\circ$	$\alpha_M=38.7922^\circ$
	$\eta=0.8026$	$\eta=0.7337$	$\eta=0.9028$	$\eta=0.8935$	$\eta=0.7670$
$z_1=10$	$z_2=60$		$z_1=90$	$z_2=540$	
$\alpha_0=20^\circ$?	$\alpha_0=26^\circ$	$\alpha_0=30^\circ$	$\alpha_0=9^\circ$	$\alpha_0=20^\circ$	$\alpha_0=30^\circ$
$\alpha_m=-11.12^\circ$?	$\alpha_m=0.6054^\circ$	$\alpha_m=7.7391^\circ$	$\alpha_m=1.3645^\circ$	$\alpha_m=16.4763^\circ$	$\alpha_m=27.7383^\circ$
$\alpha_M=38.4568^\circ$	$\alpha_M=41.4966^\circ$	$\alpha_M=43.8060^\circ$	$\alpha_M=14.9354^\circ$	$\alpha_M=23.1812^\circ$	$\alpha_M=32.0917^\circ$
	$\eta=0.8403$	$\eta=0.7908$	$\eta=0.9754$	$\eta=0.8841$	$\eta=0.7509$

In the table 4, the teeth are bended ($\beta\neq0$). The module i, take now the value 2.

Table 4. *The determination of the gear's parameters in bend teeth for i=-4*

$i_{12}=-4$	bend teeth	$\beta=15^0$			
$z_1=8$	$z_2=32$		$z_1=30$	$z_2=120$	
$\alpha_0=20^\circ$?	$\alpha_0=30^\circ$	$\alpha_0=35^\circ$	$\alpha_0=13^\circ$	$\alpha_0=20^\circ$	$\alpha_0=30^\circ$
$\alpha_m=-16.836^\circ$?	$\alpha_m=1.1265^\circ$	$\alpha_m=9.4455^\circ$	$\alpha_m=1.0269^\circ$	$\alpha_m=8.8602^\circ$	$\alpha_m=22.1550^\circ$
$\alpha_M=41.0834^\circ$	$\alpha_M=46.2592^\circ$	$\alpha_M=49.2953^\circ$	$\alpha_M=25.1344^\circ$	$\alpha_M=28.4591^\circ$	$\alpha_M=36.2518^\circ$
	$\eta=0.8046$	$\eta=0.7390$	$\eta=0.9357$	$\eta=0.8899$	$\eta=0.7593$
$z_1=18$	$z_2=72$		$z_1=90$	$z_2=360$	
$\alpha_0=19^\circ$	$\alpha_0=20^\circ$	$\alpha_0=30^\circ$	$\alpha_0=9^\circ$	$\alpha_0=20^\circ$	$\alpha_0=30^\circ$
$\alpha_m=0.32715^\circ$	$\alpha_m=2.0283^\circ$	$\alpha_m=17.1840^\circ$	$\alpha_m=1.3187^\circ$	$\alpha_m=15.8944^\circ$	$\alpha_m=26.9403^\circ$
$\alpha_M=31.7180^\circ$	$\alpha_M=32.3202^\circ$	$\alpha_M=39.1803^\circ$	$\alpha_M=14.9648^\circ$	$\alpha_M=23.6366^\circ$	$\alpha_M=32.8262^\circ$
$\eta=0.9029$	$\eta=0.8938$	$\eta=0.7702$	$\eta=0.9754$	$\eta=0.8845$	$\eta=0.7513$

2.5 Discussion and conclusions

The efficiency (of the gear) increases when the number of teeth for the driving wheel 1, z_1, increases too and when the pressure angle, α_0, diminishes; z_2 or i_{12} are not so much influence about the efficiency value;

It can easily see that for the value $\alpha_0=20^0$, the efficiency takes roughly the value $\eta \approx 0.89$ for any values of the others parameters (this justifies the choice of this value, $\alpha_0=20^0$, for the standard pressure angle of reference).

The better efficiency may be obtained only for a $\alpha_0 \neq 20^0$.

But the pressure angle of reference, α_0, can be decreased the same time the number of teeth for the driving wheel 1, z_1, increases, to increase the gears' efficiency;

Contrary, when we desire to create a gear with a low z_1 (for a less gauge), it will be necessary to increase the α_0 value, for maintaining a positive value for α_m (in this case the gear efficiency will be diminished);

When β increases, the efficiency, η, increases too, but the growth is insignificant.

The module of the gear, m, has not any influence on the gear's efficiency value.

When α_0 is diminished it can take a higher normal module, for increasing the addendum of teeth, but the increase of the module m at the same time with the increase of the z_1 can lead to a greater gauge.

The gears' efficiency, η, is really a function of α_0 and z_1: $\eta=f(\alpha_0,z_1)$; α_m and α_M are just the intermediate parameters.

For a good projection of the gear, it's necessary a z_1 and a z_2 greater than 30-60; but this condition may increase the gauge of mechanism.

In this chapter it determines precisely, the dynamics-efficiency, but at the gears transmissions, the dynamics efficiency is the same like the mechanical efficiency; this is a greater advantage of the gears transmissions. This advantage, specifically of the gear's mechanisms, may be found at the cam mechanisms with plate followers as well.

3. EFFICIENCY IN RAPPORT WITH THE CONTACT RATIO

In this paragraph one presents shortly an original method to obtain the efficiency of the geared transmissions in function of the contact ratio. With the presented relations it can make the dynamic synthesis of the geared transmissions having in view increasing the efficiency of gearing mechanisms in work [4].

3.1 Determining of gearing efficiency, in function of the contact ratio

The equation (41) is a two degree equation in x; One determines directly, Δ (42-43) and $X_{1,2}$ (44), [3].

We calculate the efficiency of a geared transmission, having in view the fact that at one moment there are several couples of teeth in contact, and not just one.

The start model has got four pairs of teeth in contact (4 couples) concomitantly.

The first couple of teeth in contact has the contact point i, defined by the ray r_{i1}, and the pressure angle α_{i1}; the forces which act at this point are: the motor force F_{mi}, perpendicular to the position vector r_{i1} at i and the force transmitted from the wheel 1 to the wheel 2 through the point i, $F_{\tau i}$, parallel to the path of action and with the sense from the wheel 1 to the wheel 2, the transmitted force being practically the projection of the motor force on the path of action; the defined velocities are similar to the forces (having in view the original kinematics, or the precise kinematics adopted); the same parameters will be defined for the next three points of contact, j, k, l (Fig. 2).

For starting we write the relations between the velocities (3.1):

$$\begin{cases} v_{\tau i} = v_{mi} \cdot \cos\alpha_i = r_i \cdot \omega_1 \cdot \cos\alpha_i = r_{b1} \cdot \omega_1 \\ v_{\tau j} = v_{mj} \cdot \cos\alpha_j = r_j \cdot \omega_1 \cdot \cos\alpha_j = r_{b1} \cdot \omega_1 \\ v_{\tau k} = v_{mk} \cdot \cos\alpha_k = r_k \cdot \omega_1 \cdot \cos\alpha_k = r_{b1} \cdot \omega_1 \\ v_{\tau l} = v_{ml} \cdot \cos\alpha_l = r_l \cdot \omega_1 \cdot \cos\alpha_l = r_{b1} \cdot \omega_1 \end{cases} \quad (3.1)$$

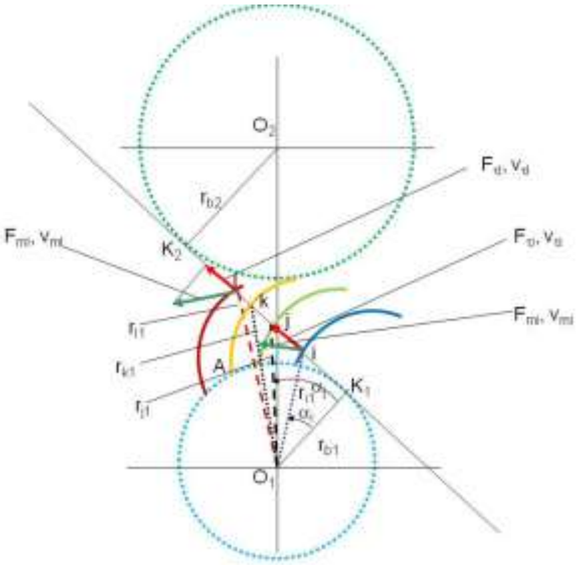


Fig. 2. *Four pairs of teeth in contact concomitantly*

From relations (3.1), one obtains the equality of the tangential velocities (3.2), and makes explicit the motor velocities (3.3).

$$v_{\tau i} = v_{\tau j} = v_{\tau k} = v_{\tau l} = r_{b1} \cdot \omega_1 \quad (3.2)$$

$$\begin{cases} v_{mi} = \dfrac{r_{b1} \cdot \omega_1}{\cos\alpha_i}; v_{mj} = \dfrac{r_{b1} \cdot \omega_1}{\cos\alpha_j}; \\ v_{mk} = \dfrac{r_{b1} \cdot \omega_1}{\cos\alpha_k}; v_{ml} = \dfrac{r_{b1} \cdot \omega_1}{\cos\alpha_l} \end{cases} \quad (3.3)$$

The forces transmitted concomitantly at the four points must be the same (3.4):

$$F_{\tau i} = F_{\tau j} = F_{\tau k} = F_{\tau l} = F_\tau \quad (3.4)$$

The motor forces are (3.5):

$$\begin{cases} F_{mi} = \dfrac{F_\tau}{\cos\alpha_i}; F_{mj} = \dfrac{F_\tau}{\cos\alpha_j}; \\ F_{mk} = \dfrac{F_\tau}{\cos\alpha_k}; F_{ml} = \dfrac{F_\tau}{\cos\alpha_l} \end{cases} \quad (3.5)$$

The momentary efficiency can be written in the form (3.6).

$$\eta_i = \frac{P_u}{P_c} = \frac{P_\tau}{P_m} =$$
$$= \frac{F_{\tau i} \cdot v_{\tau i} + F_{\tau j} \cdot v_{\tau j} + F_{\tau k} \cdot v_{\tau k} + F_{\tau l} \cdot v_{\tau l}}{F_{mi} \cdot v_{mi} + F_{mj} \cdot v_{mj} + F_{mk} \cdot v_{mk} + F_{ml} \cdot v_{ml}} =$$
$$= \frac{4 \cdot F_\tau \cdot r_{b1} \cdot \omega_1}{\dfrac{F_\tau \cdot r_{b1} \cdot \omega_1}{\cos^2\alpha_i} + \dfrac{F_\tau \cdot r_{b1} \cdot \omega_1}{\cos^2\alpha_j} + \dfrac{F_\tau \cdot r_{b1} \cdot \omega_1}{\cos^2\alpha_k} + \dfrac{F_\tau \cdot r_{b1} \cdot \omega_1}{\cos^2\alpha_l}} =$$
$$= \frac{4}{\dfrac{1}{\cos^2\alpha_i} + \dfrac{1}{\cos^2\alpha_j} + \dfrac{1}{\cos^2\alpha_k} + \dfrac{1}{\cos^2\alpha_l}} =$$
$$= \frac{4}{4 + tg^2\alpha_i + tg^2\alpha_j + tg^2\alpha_k + tg^2\alpha_l} \quad (3.6)$$

Relations (3.7) and (3.8) are auxiliary (relations):

$$\begin{cases} K_1 i = r_{b1} \cdot tg\alpha_i ; K_1 j = r_{b1} \cdot tg\alpha_j ; \\ K_1 k = r_{b1} \cdot tg\alpha_k ; K_1 l = r_{b1} \cdot tg\alpha_l \\ K_1 j - K_1 i = r_{b1} \cdot (tg\alpha_j - tg\alpha_i); \\ K_1 j - K_1 i = r_{b1} \cdot \frac{2 \cdot \pi}{z_1} \Rightarrow tg\alpha_j = tg\alpha_i + \frac{2 \cdot \pi}{z_1} \\ K_1 k - K_1 i = r_{b1} \cdot (tg\alpha_k - tg\alpha_i); \\ K_1 k - K_1 i = r_{b1} \cdot 2 \cdot \frac{2 \cdot \pi}{z_1} \Rightarrow \\ \Rightarrow tg\alpha_k = tg\alpha_i + 2 \cdot \frac{2 \cdot \pi}{z_1} \\ K_1 l - K_1 i = r_{b1} \cdot (tg\alpha_l - tg\alpha_i); \\ K_1 l - K_1 i = r_{b1} \cdot 3 \cdot \frac{2 \cdot \pi}{z_1} \Rightarrow \\ \Rightarrow tg\alpha_l = tg\alpha_i + 3 \cdot \frac{2 \cdot \pi}{z_1} \end{cases} \quad (3.7)$$

$$\begin{cases} tg\alpha_j = tg\alpha_i \pm \frac{2 \cdot \pi}{z_1}; \\ tg\alpha_k = tg\alpha_i \pm 2 \cdot \frac{2 \cdot \pi}{z_1}; \\ tg\alpha_l = tg\alpha_i \pm 3 \cdot \frac{2 \cdot \pi}{z_1} \end{cases} \quad (3.8)$$

One keeps relations (3.8), with the sign plus (+) for the gearing where the driving wheel 1 has external teeth (at the external or internal gearing), and with the sign (-) for the gearing where the driving wheel 1, has internal teeth (when the driving wheel is a ring, only for the internal gearing).

The relation of the momentary efficiency (3.6) uses the auxiliary relations (3.8) and takes the form (3.9).

$$\begin{aligned} \eta_i &= \frac{4}{4 + tg^2\alpha_i + tg^2\alpha_j + tg^2\alpha_k + tg^2\alpha_l} = \\ &= \frac{4}{4 + tg^2\alpha_i + (tg\alpha_i \pm \frac{2\pi}{z_1})^2 + (tg\alpha_i \pm 2 \cdot \frac{2\pi}{z_1})^2 + (tg\alpha_i \pm 3 \cdot \frac{2\pi}{z_1})^2} = \\ &= \frac{4}{4 + 4 \cdot tg^2\alpha_i + \frac{4\pi^2}{z_1^2} \cdot (0^2 + 1^2 + 2^2 + 3^2) \pm 2 \cdot tg\alpha_i \cdot \frac{2\pi}{z_1} \cdot (0 + 1 + 2 + 3)} = \\ &= \frac{1}{1 + tg^2\alpha_i + \frac{4\pi^2}{E \cdot z_1^2} \cdot \sum_{i=1}^{E} (i-1)^2 \pm 2 \cdot tg\alpha_i \cdot \frac{2\pi}{E \cdot z_1} \cdot \sum_{i=1}^{E} (i-1)} = \\ &= \frac{1}{1 + tg^2\alpha_1 + \frac{4\pi^2}{E \cdot z_1^2} \cdot \frac{E \cdot (E-1) \cdot (2 \cdot E - 1)}{6} \pm \frac{4\pi \cdot tg\alpha_1}{E \cdot z_1} \cdot \frac{E \cdot (E-1)}{2}} = \\ &= \frac{1}{1 + tg^2\alpha_1 + \frac{2\pi^2 \cdot (E-1) \cdot (2E-1)}{3 \cdot z_1^2} \pm \frac{2\pi \cdot tg\alpha_1 \cdot (E-1)}{z_1}} = \\ &= \frac{1}{1 + tg^2\alpha_1 + \frac{2\pi^2}{3 \cdot z_1^2} \cdot (\varepsilon_{12} - 1) \cdot (2 \cdot \varepsilon_{12} - 1) \pm \frac{2\pi \cdot tg\alpha_1}{z_1} \cdot (\varepsilon_{12} - 1)} \end{aligned} \quad (3.9)$$

In expression (3.9) one starts with relation (3.6) where four pairs are in contact concomitantly, but then one generalizes the expression, replacing the four pairs with E couples, (replacing 4 with the E variable), which represents the whole number of the contact ratio +1, and after restricting the sums expressions, we replace the variable E with the contact ratio ε_{12}, as well.

The mechanical efficiency offers more advantages than the momentary efficiency, and will be calculated approximately, by replacing in relation (3.9) the pressure angle α_1, with the normal pressure angle α_0 the relation taking the form (3.10); where ε_{12} represents the contact ratio of the gearing, and it will be calculated with expression (3.11) for the external gearing, and with relation (3.12) for the internal gearing.

$$\eta_m = \frac{1}{1 + tg^2\alpha_0 + \frac{2\pi^2}{3 \cdot z_1^2} \cdot (\varepsilon_{12} - 1) \cdot (2 \cdot \varepsilon_{12} - 1) \pm \frac{2\pi \cdot tg\alpha_0}{z_1} \cdot (\varepsilon_{12} - 1)} \quad (3.10)$$

$$\varepsilon_{12}^{a.e.} = \frac{\sqrt{z_1^2 \cdot \sin^2 \alpha_0 + 4 \cdot z_1 + 4} + \sqrt{z_2^2 \cdot \sin^2 \alpha_0 + 4 \cdot z_2 + 4} - (z_1 + z_2) \cdot \sin \alpha_0}{2 \cdot \pi \cdot \cos \alpha_0} \tag{3.11}$$

$$\varepsilon_{12}^{a.i.} = \frac{\sqrt{z_e^2 \cdot \sin^2 \alpha_0 + 4 \cdot z_e + 4} - \sqrt{z_i^2 \cdot \sin^2 \alpha_0 - 4 \cdot z_i + 4} + (z_i - z_e) \cdot \sin \alpha_0}{2 \cdot \pi \cdot \cos \alpha_0} \tag{3.12}$$

The made calculations have been centralized in the table 5.

Table 5. *Right teeth, β=0 [deg].*

Table 5								
The centralized results								
z1	α_0 [grad]	z2	ε_{12}^{ae}	η_{12}^{ae}	η_{21}^{ae}	ε_{12}^{ai}	η_{12}^{ai}	η_{21}^{ai}
42	20	126	1.79	**0.844**	**0.871**	1.92	**0.838**	**0.895**
46	19	138	1.87	**0.856**	**0.882**	2.00	**0.850**	**0.905**
52	18	156	1.96	**0.869**	**0.893**	2.09	**0.864**	**0.915**
58	17	174	2.06	**0.880**	**0.904**	2.20	**0.876**	**0.925**
65	16	195	2.17	**0.892**	**0.914**	2.32	**0.887**	**0.933**
74	15	222	2.30	**0.903**	**0.923**	2.46	**0.899**	**0.942**
85	14	255	2.44	**0.914**	**0.933**	2.62	**0.910**	**0.949**
98	13	294	2.62	**0.924**	**0.941**	2.81	**0.920**	**0.956**
115	12	345	2.82	**0.934**	**0.949**	3.02	**0.931**	**0.963**
137	11	411	3.06	**0.943**	**0.957**	3.28	**0.941**	**0.969**
165	10	495	3.35	**0.952**	**0.964**	3.59	**0.950**	**0.974**
204	9	510	3.68	**0.960**	**0.970**	4.02	**0.958**	**0.980**
257	8	514	4.09	**0.968**	**0.975**	4.57	**0.966**	**0.985**
336	7	672	4.66	**0.975**	**0.980**	5.21	**0.973**	**0.989**
457	6	914	5.42	**0.981**	**0.985**	6.06	**0.980**	**0.992**
657	5	1314	6.49	**0.986**	**0.989**	7.26	**0.986**	**0.994**

3.2 Determining of gearing efficiency, in function of the contact ratio, to the bended teeth

Generally we use gearings with teeth inclined (with bended teeth). For gears with bended teeth, the calculations show a decrease in yield when the inclination angle increases. For angles with inclination which not exceed 25 degree the efficiency of gearing is good (see the table 6).

Table 6. *Bended teeth, β=25 [deg].*

Table 6 Bended teeth, β=25 [deg]								
Determining the efficiency when *β=25 [deg]*								
z1	α_0 [grad]	z2	ε_{12}^{ae}	η_{12}^{ae}	η_{21}^{ae}	ε_{12}^{ai}	η_{12}^{ai}	η_{21}^{ai}
42	20	126	1,708	0,829	0,851	1,791	0,826	0,871
46	19	138	1,776	0,843	0,864	1,865	0,839	0,883
52	18	156	1,859	0,856	0,876	1,949	0,853	0,895
58	17	174	1,946	0,869	0,889	2,043	0,866	0,906
65	16	195	2,058	0,882	0,900	2,151	0,879	0,917
74	15	222	2,165	0,894	0,911	2,275	0,892	0,927
85	14	255	2,299	0,906	0,922	2,418	0,904	0,936
98	13	294	2,456	0,917	0,932	2,584	0,915	0,945
115	12	345	2,641	0,928	0,941	2,780	0,926	0,953
137	11	411	2,863	0,938	0,950	3,013	0,937	0,961
165	10	495	3,129	0,948	0,958	3,295	0,947	0,968
204	9	510	3,443	0,957	0,965	3,665	0,956	0,974
257	8	514	3,829	0,965	0,971	4,146	0,964	0,981
336	7	672	4,357	0,973	0,977	4,719	0,972	0,985
457	6	914	5,064	0,980	0,983	5,486	0,979	0,989
657	5	1314	6,056	0,985	0,988	6,563	0,985	0,992

When the inclination angle (β) exceeds 25 degrees the gearing will suffer a significant drop in yield (see the tables 7-8).

Table 7. *Bended teeth, β=35 [deg].*

Table 7 Bended teeth, β=35 [deg]								
Determining the efficiency when *β=35 [deg]*								
z1	α_0 [grad]	z2	ε_{12}^{ae}	η_{12}^{ae}	η_{21}^{ae}	ε_{12}^{ai}	η_{12}^{ai}	η_{21}^{ai}
42	20	126	1,620	0,809	0,827	1,677	0,807	0,843
46	19	138	1,681	0,825	0,841	1,741	0,822	0,858
52	18	156	1,755	0,840	0,856	1,815	0,838	0,871
58	17	174	1,832	0,854	0,870	1,898	0,852	0,885
65	16	195	1,948	0,868	0,883	1,993	0,867	0,897
74	15	222	2,030	0,882	0,896	2,103	0,881	0,909
85	14	255	2,150	0,895	0,909	2,230	0,894	0,921
98	13	294	2,293	0,908	0,920	2,379	0,907	0,932
115	12	345	2,461	0,920	0,931	2,554	0,919	0,942
137	11	411	2,663	0,932	0,942	2,764	0,931	0,951
165	10	495	2,906	0,942	0,951	3,017	0,942	0,959
204	9	510	3,196	0,952	0,959	3,345	0,952	0,968
257	8	514	3,556	0,962	0,967	3,766	0,961	0,975
336	7	672	4,041	0,970	0,974	4,281	0,969	0,981
457	6	914	4,692	0,978	0,981	4,971	0,977	0,986
657	5	1314	5,607	0,984	0,986	5,942	0,984	0,990

Table 8. *Bended teeth, β=45 [deg].*

Table 8 Bended teeth, β=45 [deg]								
Determining the efficiency when *β=45 [deg]*								
$z1$	α_0 [grad]	$z2$	$\varepsilon_{12}^{a.e.}$	$\eta_{12}^{a.e.}$	$\eta_{21}^{a.e.}$	$\varepsilon_{12}^{a.i.}$	$\eta_{12}^{a.i.}$	$\eta_{21}^{a.i.}$
42	20	126	1,505	0,772	0,784	1,539	0,771	0,796
46	19	138	1,555	0,790	0,802	1,590	0,789	0,814
52	18	156	1,618	0,808	0,820	1,650	0,807	0,831
58	17	174	1,680	0,825	0,837	1,718	0,824	0,848
65	16	195	1,810	0,841	0,853	1,796	0,841	0,864
74	15	222	1,848	0,858	0,869	1,888	0,858	0,879
85	14	255	1,949	0,874	0,884	1,994	0,874	0,894
98	13	294	2,070	0,889	0,899	2,119	0,889	0,908
115	12	345	2,215	0,904	0,913	2,268	0,903	0,921
137	11	411	2,389	0,918	0,926	2,446	0,917	0,933
165	10	495	2,600	0,931	0,938	2,662	0,930	0,944
204	9	510	2,855	0,943	0,948	2,938	0,943	0,955
257	8	514	3,173	0,954	0,958	3,290	0,954	0,965
336	7	672	3,599	0,964	0,967	3,732	0,964	0,973
457	6	914	4,171	0,973	0,976	4,325	0,973	0,980
657	5	1314	4,976	0,981	0,983	5,161	0,981	0,986

New calculation relationships can be put in the forms (3.13-3.15).

$$\eta_m = \frac{z_1^2 \cdot \cos^2 \beta}{z_1^2 (tg^2 \alpha_0 + \cos^2 \beta) + \frac{2}{3}\pi^2 \cos^4 \beta (\varepsilon - 1)(2\varepsilon - 1) \pm 2\pi tg \alpha_0 z_1 \cos^2 \beta (\varepsilon - 1)} \tag{3.13}$$

$$\varepsilon^{a.e.} = \frac{1 + tg^2 \beta}{2 \cdot \pi} \cdot \left\{ \sqrt{\left[(z_1 + 2 \cdot \cos\beta) \cdot tg\alpha_0\right]^2 + 4 \cdot \cos^3 \beta \cdot (z_1 + \cos\beta)} + \sqrt{\left[(z_2 + 2 \cdot \cos\beta) \cdot tg\alpha_0\right]^2 + 4 \cdot \cos^3 \beta \cdot (z_2 + \cos\beta)} - (z_1 + z_2) \cdot tg\alpha_0 \right\} \tag{3.14}$$

$$\varepsilon^{a.i.} = \frac{1 + tg^2 \beta}{2 \cdot \pi} \cdot \left\{ \sqrt{\left[(z_e + 2 \cdot \cos\beta) \cdot tg\alpha_0\right]^2 + 4 \cdot \cos^3 \beta \cdot (z_e + \cos\beta)} - \sqrt{\left[(z_i - 2 \cdot \cos\beta) \cdot tg\alpha_0\right]^2 - 4 \cdot \cos^3 \beta \cdot (z_i - \cos\beta)} - (z_e - z_i) \cdot tg\alpha_0 \right\} \tag{3.15}$$

The calculation relationships (3.13-3.15) are general. They have the advantage that can be used with great precision in determining the efficiency of any type of gearings.

To use them at the gearing without bended teeth is enough to assign them a beta value = zero. The results obtained in this case will be identical to the ones of the relations 3.10-3.12.

3.3 Discussion and conclusions

The best efficiency can be obtained with the internal gearing when the drive wheel 1 is the ring; the minimum efficiency will be obtained when the drive wheel 1 of the internal gearing has external teeth.

For the external gearing, the best efficiency is obtained when the bigger wheel is the drive wheel; *when we decrease the normal angle α_0, the contact ratio increases and the efficiency increases as well.*

The efficiency increases too, when the number of teeth of the drive wheel 1 increases (when z_1 increases).

4. AVOID THE INTERFERENCE

At the gear mechanisms an important problem is the interference of the teeth. To avoid the interference between teeth, we must know the minimum number of teeth of the driving wheel, in function of the pressure angle (normal on the pitch circle, alpha0), in function of the tooth inclination angle (beta), and in function transmission ratio (i). The chapter presents an original method to make the geometric synthesis of the gear, having in view the minimum number of teeth of the driving wheel.

The classical methods use many different relations to determine the minimum number of teeth of the driving wheel [5-8].

By this paper we want to give a unitary method to determine the minimum number of teeth of the driving wheel 1, to avoid the interference between the teeth of the two wheels (of the gear) [7].

The basic condition of interference, is the same, but the originality of this new presented method consist in the mode in which it was solved the classical relationship.

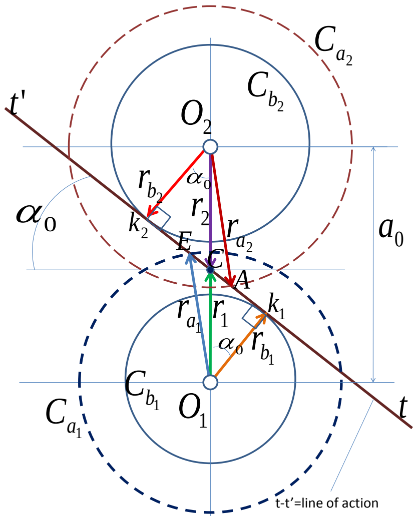
The minimum number of teeth of the driving wheel 1, is a function on some parameters: the pressure angle (normal on the pitch circle, α_0), the tooth inclination angle (β), and the transmission ratio ($i=|i_{12}|=|-z_2/z_1|=z_2/z_1$).

4.1 Avoid the phenomenon of interference

In order to avoid interference phenomenon, point A must lie between C and K_1 (the addendum circle of the wheel 2, C_{a2} need to cut the line of action between points C and K_1, and under no circumstances does not exceed the point K_1). Similarly, C_{a1} addendum circle must cut the action line between points C and K_2, resulting point E, which in no circumstances, does not exceed the point K_2.

The conditions to avoid the phenomenon of interference can be written with the relations (4.1).

The basic conditions of interference, are the same ($CA<K_1C$; $CE<K_2C$), but the originality of this new presented method consist in the mode in which it was solved the classical relationship (see the system 4.1) [7], (see the Fig. 3).



$A=C_{a_2}\cap tt'$; $E=C_{a_1}\cap tt'$ $\Rightarrow$

$\Rightarrow AE = segment\ \ gear$

Fig. 3 *Line of action (t-t')*

$$\begin{cases}
CA<K_1C \quad and \quad CE<K_2C \\
CA=K_2A-K_2C=\sqrt{r_{a_2}^2-r_{b_2}^2}-r_2\cdot\sin\alpha_0;\quad CA<K_1C\Rightarrow \\
\Rightarrow\sqrt{r_{a_2}^2-r_{b_2}^2}-r_2\cdot\sin\alpha_0<r_1\cdot\sin\alpha_0\Rightarrow\sqrt{r_{a_2}^2-r_{b_2}^2}<(r_1+r_2)\cdot\sin\alpha_0 \\
\Rightarrow d_{a_2}^2-d_{b_2}^2<(d_1+d_2)^2\cdot\sin^2\alpha_0\Rightarrow \\
\Rightarrow m^2\cdot(z_2+2)^2-m^2\cdot z_2^2\cdot\cos^2\alpha_0<m^2\cdot(z_1+z_2)^2\cdot\sin^2\alpha_0\Rightarrow \\
\Rightarrow z_2^2+4\cdot z_2+4-z_2^2<z_1^2\cdot\sin^2\alpha_0+2\cdot z_1\cdot z_2\cdot\sin^2\alpha_0\Rightarrow \\
\Rightarrow 4\cdot z_2+4<z_1^2\cdot\sin^2\alpha_0+2\cdot z_1\cdot z_2\cdot\sin^2\alpha_0 \\
from \quad CE<K_2C\Rightarrow 4\cdot z_1+4<z_2^2\cdot\sin^2\alpha_0+2\cdot z_1\cdot z_2\cdot\sin^2\alpha_0 \\
it \ obtains \ the \ system\begin{cases}4\cdot z_2+4<z_1^2\cdot\sin^2\alpha_0+2\cdot z_1\cdot z_2\cdot\sin^2\alpha_0\\ 4\cdot z_1+4<z_2^2\cdot\sin^2\alpha_0+2\cdot z_1\cdot z_2\cdot\sin^2\alpha_0\end{cases} \\
take \quad i\equiv|i_{12}|=\frac{z_2}{z_1}\Rightarrow z_2=i\cdot z_1; result \ the \ system \\
\begin{cases}\sin^2\alpha_0\cdot(1+2\cdot i)\cdot z_1^2-2\cdot2\cdot i\cdot z_1-4>0\\ \sin^2\alpha_0\cdot(i^2+2\cdot i)\cdot z_1^2-2\cdot2\cdot z_1-4>0\end{cases} \quad with \ the \ solutions: \\
\begin{cases}z_{1_{1,2}}=\frac{2\cdot i\pm2\cdot\sqrt{i^2+\sin^2\alpha_0+2\cdot i\cdot\sin^2\alpha_0}}{(2\cdot i+1)\cdot\sin^2\alpha_0}\\ z_{1_{3,4}}=\frac{2\pm2\cdot\sqrt{1+i^2\cdot\sin^2\alpha_0+2\cdot i\cdot\sin^2\alpha_0}}{(2\cdot i+i^2)\cdot\sin^2\alpha_0}\end{cases} it \ keeps \ solutions \ + \\
\begin{cases}z_{1_2}=2\cdot\frac{i+\sqrt{i^2+\sin^2\alpha_0+2\cdot i\cdot\sin^2\alpha_0}}{(2\cdot i+1)\cdot\sin^2\alpha_0}\\ z_{1_4}=2\cdot\frac{1+\sqrt{1+i^2\cdot\sin^2\alpha_0+2\cdot i\cdot\sin^2\alpha_0}}{(2\cdot i+i^2)\cdot\sin^2\alpha_0}\end{cases}
\end{cases} \tag{4.1}$$

Relationship which generates z_{1_4} always gives lower values than the relationship which generates z_{1_2} so it is sufficient the condition (4.2) for finding the minimum number of teeth of the wheel 1, necessary to avoid interference [7].

$$z_{\min} \equiv z_{1_2} = $$
$$= 2 \cdot \frac{i + \sqrt{i^2 + \sin^2 \alpha_0 + 2 \cdot i \cdot \sin^2 \alpha_0}}{(2 \cdot i + 1) \cdot \sin^2 \alpha_0} \tag{4.2}$$

When we have inclined teeth, one takes $z_{min} \rightarrow z_{min}/\cos\beta$, and $\alpha_0 \rightarrow \alpha_{0t}$, and the relationship (4.2) takes the form (4.3).

The minimum number of teeth of the driving wheel 1, is a function on some parameters: the pressure angle (normal on the pitch circle, α_0), the tooth inclination angle (β), and the transmission ratio ($i=|i_{12}|=|-z_2/z_1|=z_2/z_1$), (see the relationship 4.3, and [5], [8]).

$$\begin{cases} z_{\min} \equiv z_{1_2} = 2 \cdot \cos\beta \cdot \\ \cdot \dfrac{i + \sqrt{i^2 + \sin^2 \alpha_{0t} + 2 \cdot i \cdot \sin^2 \alpha_{0t}}}{(2 \cdot i + 1) \cdot \sin^2 \alpha_{0t}} \\ where: \quad tg\alpha_{0t} = \dfrac{tg\alpha_0}{\cos\beta} \Rightarrow \\ \Rightarrow \alpha_{0t} = arctg\left(\dfrac{tg\alpha_0}{\cos\beta}\right) \end{cases} \tag{4.3}$$

The system (4.3) is a simple, unitar and general relationship which can give the solutions of the minimum number of teeth of the wheel 1 (the driving wheel), to avoid the interference.

In the following tables (9-23) is chosen an alpha0 value (35 [deg]), and successively increased beta angle values (from 0 [deg] to 40 [deg]) and the transmission ratio i (from 1, to 80), and one gets the minimum numbers of teeth. Then, we will decrease successively the value of the angle alpha0 (from 35 [deg] to 5 [deg]). See the tables (9-23).

Table 9 α_0=35 [deg], β=0 [deg]

α_0 [deg]	35		β [deg]	0						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	4.8828	5.0325	5.1896	5.3204	5.4386	5.5467	5.6431	5.7198	5.7867	5.8439
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	5.8880	5.9243	5.9568	5.9805	5.9997	6.0158	6.0290	6.0389	6.0471	6.0539

Table 10 α_0=35 [deg], β=10 [deg]

α_0 [deg]	35		β [deg]	10						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	4.7254	4.8679	5.0174	5.1419	5.2545	5.3576	5.4495	5.5226	5.5865	5.6411
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	5.6832	5.7178	5.7489	5.7715	5.7898	5.8052	5.8178	5.8273	5.8351	5.8416

Table 11 α_0=35 [deg], β=20 [deg]

α_0 [deg]	35		β [deg]	20						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	4.2799	4.4022	4.5307	4.6379	4.7351	4.8240	4.9035	4.9667	5.0219	5.0693
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	5.1058	5.1358	5.1627	5.1824	5.1983	5.2116	5.2226	5.2308	5.2376	5.2432

Table 12 α_0=35 [deg], β=30 [deg]

α_0 [deg]	35		β [deg]	30						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	3.6198	3.7136	3.8123	3.8949	3.9699	4.0388	4.1004	4.1495	4.1925	4.2294
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	4.2578	4.2812	4.3022	4.3176	4.3300	4.3404	4.3490	4.3554	4.3608	4.3651

Table 13 α_0=35 [deg], β=40 [deg]

α_0 [deg]	35		β [deg]	40						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	2.8475	2.9104	2.9769	3.0322	3.0836	3.1304	3.1724	3.2060	3.2355	3.2608
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	3.2804	3.2965	3.3110	3.3216	3.3302	3.3374	3.3433	3.3477	3.3514	3.3545

Table 14 α_0=20 [deg], β=0 [deg]

α_0 [deg]	20		β [deg]	0						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	12.323	12.966	13.624	14.161	14.637	15.066	15.443	15.740	15.997	16.215
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	16.382	16.519	16.641	16.730	16.802	16.862	16.911	16.948	16.978	17.003

Table 15 α_0=20 [deg], β=10 [deg]

α_0 [deg]	20		β [deg]	10						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	11.835	12.447	13.075	13.586	14.041	14.450	14.810	15.094	15.339	15.547
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	15.706	15.837	15.954	16.039	16.107	16.164	16.211	16.246	16.275	16.299

Table 16 α_0=20 [deg], β=20 [deg]

α_0 [deg]	20		β [deg]	20						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	10.446	10.994	11.535	11.977	12.370	12.725	13.036	13.282	13.495	13.676
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	13.814	13.927	14.028	14.102	14.161	14.211	14.252	14.282	14.308	14.328

Table 17 α_0=20 [deg], β=30 [deg]

α_0 [deg]	20		β [deg]	30						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	8.4777	8.8841	9.3024	9.6448	9.9497	10.225	10.468	10.659	10.825	10.966
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	11.074	11.163	11.242	11.299	11.346	11.385	11.417	11.441	11.461	11.477

Table 18 α_0=20 [deg], β=40 [deg]

α_0 [deg]	20		β [deg]	40						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	6.2280	6.5020	6.7854	7.0182	7.2263	7.4147	7.5813	7.7128	7.8269	7.9241
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	7.9985	8.0597	8.1143	8.1540	8.1862	8.2131	8.2352	8.2517	8.2654	8.2767

Table 19 α_0=5 [deg], β=0 [deg]

α_0 [deg]	5		β [deg]	0						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	176.52	188.86	201.22	211.13	219.81	227.54	234.28	239.55	244.09	247.93
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	250.85	253.24	255.37	256.92	258.17	259.21	260.06	260.70	261.23	261.67

Table 20 α_0=5 [deg], β=10 [deg]

α_0 [deg]	5		β [deg]	10						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	168.66	180.45	192.25	201.72	210.01	217.38	223.83	228.86	233.19	236.86
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	239.65	241.94	243.97	245.45	246.64	247.63	248.45	249.06	249.57	249.98

Table 21 α_0=5 [deg], β=20 [deg]

α_0 [deg]	5		β [deg]	20						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	146.73	156.95	167.20	175.42	182.62	189.03	194.63	198.99	202.76	205.94
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	208.37	210.35	212.12	213.40	214.44	215.30	216.01	216.54	216.98	217.34

Table 22 α_0=5 [deg], β=30 [deg]

α_0 [deg]	5		β [deg]	30						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	115.16	123.15	131.16	137.58	143.22	148.23	152.61	156.03	158.97	161.47
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	163.37	164.92	166.30	167.31	168.12	168.79	169.35	169.76	170.11	170.39

Table 23 α_0=5 [deg], β=40 [deg]

α_0 [deg]	5		β [deg]	40						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	80.086	85.602	91.136	95.575	99.465	102.93	105.96	108.33	110.36	112.09
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	113.40	114.48	115.43	116.13	116.69	117.16	117.54	117.83	118.07	118.26

4.2 Discussion and conclusions

The presented method has the great advantage to optimize the number of teeth for a gear before to make its synthesis. In this mode the constructor may elect the minimum number of teeth, for an imposed transmission ratio, i.

Classical to realize an i=2, the constructor can select between 18 or 33 teeth to the driving wheel 1, which means a 36 or 66 teeth for the driven wheel 2. With the aid of the presented tables, he can make a multiple selection.

The engineer can select for the driving wheel 1 a number of teeth z_1=6, with an $alpha_0$=35 [deg], and a beta=0 [deg]. He may do this not only for a transmission ratio i=2, but and for the domain from i=1.25 to i=25 (see the table 9).

If he elect $alpha_0$=35 [deg] and beta=40 [deg], then he can take a number of teeth for the driving wheel 1 of z_1=4, and can do this for the entire domain from i=2 to i=80 (see the table 13). The constructor may do this when it is necessary a minimum number of teeth, but with an efficiency of the gear decreasing.

Contrary, when we wish a great efficiency, one must increase the number of teeth and decrease the angles $alpha_0$ and beta. With an $alpha_0$=5 [deg] and beta=0 (table 19), we can take the number of teeth at the driving wheel 1, from z_1=189 (for i=1.25), to z_1=262 (for i=80).

For the known classical $alpha_0$=20 [deg] and beta=0 [deg], when the ratio i vary from 1 to 80, z_1 takes the values from 13 to 18 (see the table 14). With the classical method it was taken only the minimum value 18 to the minimum number of teeth [7].

5. CONCLUSIONS

The presented method manages to synthesize (in theory) the best option parameters for any desired gear.

Relationships shown have the great advantage of donating optimal solutions for any situation you want without the need for difficult calculations, experimental building, or specialized software.

Comparisons made with specialized software (Inventor) showed a precision (matching) perfect. Workload and procedures could be so much smaller.

The parallel drawn between the software "Autodesk Inventor" and the presented calculation relationships will be highlighted in the following paper (as handle a large volume), [13].

Applied in the automotive industry, at the transmission mechanisms, these changes may decrease overall fuel consumption further, and pollutant emissions [15].

Gears can be designed to operate without noise (see [1], [4-5], [14-15]).

But, the applications will be spectacular in the automatic transmissions used in aerospace, in robotics and mechatronics.

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