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Direct and inverse kinematics to the anthropomorphic robots

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Abstract: The paper presents an original geometrical and kinematic method for the study of geometry and determining positions of a MP-3R structure. It presents shortly the MP-3R direct and inverse kinematics, the inverse kinematics being solved by an original exactly method. One presents shortly an original method to solve the robot inverse kinematics exemplified at the 3R-Robots (MP-3R). The system which must be solved has three equations and three independent parameters to determine. Constructive basis is represented by a robot with three degrees of freedom (a robot with three axes of rotation). If one study (analyzes) an anthropomorphic robot with three axes of rotation (which represents the main movements, absolutely necessary), it already has a base system, on which one can then add other movements (secondary, additional). Calculations were arranged and in the matrix form.

Keywords: Anthropomorphic robots, Direct kinematics, Inverse kinematics, 3R systems, Matrix systems.

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1. Introduction

Although the anthropomorphic robots, have different structural forms, in recent years have been developed especially those with rotating movements, with three or more axis. Constructive basis is represented by a robot with three degrees of freedom (a robot with three axes of rotation) [1]. If we study (analyze) an anthropomorphic robot with three axes of rotation (which represents the main movements, absolutely necessary), we already have a base system, on which we can then add other movements (secondary, additional). The base system has three rotary axes: a vertical axis (by this axis all the system is rotated, for positioning), and two horizontal axes (each making possible a rotation of an arm). Calculations were arranged and in the matrix form.

In direct kinematics, known kinematic parameters (input parameters) are absolute rotation angles of the three mobile elements: φ_{10}, φ_{20}, φ_{30}, the rotation angles of the three actuators (electric motors, mounted in the rotational kinematic couplings), and the determined parameters (output parameters) are the three absolute coordinates x_M, y_M, z_M of the point M, ie kinematic parameters (coordinates) of the endeffector (which can be a hand, to grabbed, a soldering tip, painted, cut, etc).

2. Geometry and Direct Kinematic, to the MP-3R

Kinematics of serial manipulators and robots will be illustrated by a 3R kinematic model (see Fig. 01), a medium difficulty system, ideal for understanding the phenomenon, but also to specify the basic knowledge necessary for starting calculations for systems simpler and more complex.

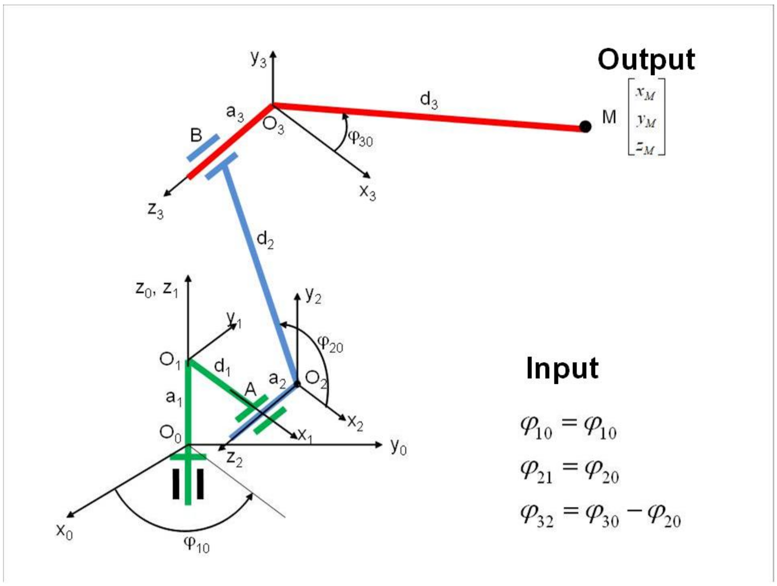

Fig. 1 *Geometry and direct kinematics to a MP-3R*

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Fixed coordinate system was noted with $x_0O_0y_0z_0$. Mobile systems related to (reinforced by) the three mobile elements (1, 2, 3) have indices 1, 2 and 3. Their orientation was chosen conveniently. Known kinematic parameters (input parameters in direct kinematics) are absolute rotation angles of the three mobile elements: φ_{10}, φ_{20}, φ_{30}, the rotation angles of the three actuators (electric motors, mounted in the rotational kinematic couplings). Determined parameters (output parameters) are the three absolute coordinates x_M, y_M, z_M of the point M, ie kinematic parameters (coordinates) of the endeffector (which can be a hand, to grabbed, a soldering tip, painted, cut, etc).

To begin one writes vector matrix (A_{01}) which change the coordinates of the origin of the coordinate system, by linear moving (displacement) from O_0 to O_1, when the axes remain parallel to each other permanently (see Eq. 2.1).

$$A_{01} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \tag{2.1}$$

Next we write the rotation matrix T_{01}, which rotates system $x_1O_1y_1z_1$ in rapport with the system $x_0O_0y_0z_0$ (it is a 3x3 square matrix; see the relationship 2.2).

$$\begin{aligned} T_{01} &= \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \\ &= \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{2.2}$$

On the first column (which represents the coordinates of the rotated axis O_1x_1) it writes the coordinates of the unit vector of O_1x_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O1 but without rotation; see the relationship 2.3).

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \tag{2.3}$$

ISSN: 1415-7314
ISSN online: 2317-6717

On the second column of the matrix T_{01} it writes the coordinates of the unit vector of the rotated axis O_1y_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O_1 but without rotation system; see the relationship 2.4).

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} \tag{2.4}$$

On the third column of the matrix T_{01} it writes the coordinates of the unit vector of the rotated axis O_1z_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O_1 but without rotation system; see the relationship 2.5).

$$\begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \end{bmatrix} \tag{2.5}$$

In the elected case (figure 1), the unit vector of the rotated axis O_1x_1, has in rapport of the old system $x_0O_0y_0z_0$, translated into O_1 without rotation, the coordinates given by the column unit vector (relationship 2.6).

$$\begin{bmatrix} \alpha_x = 1 \cdot \cos\varphi_{10} = \cos\varphi_{10} \\ \alpha_y = 1 \cdot \sin\varphi_{10} = \sin\varphi_{10} \\ \alpha_z = 1 \cdot \cos 90^0 = 1 \cdot 0 = 0 \end{bmatrix} \tag{2.6}$$

The unit vector of the rotated axis O_1y_1, has in rapport of the old system of axes $x_0O_0y_0z_0$ (translated into O1 without rotation), coordinates data unit vector column (relationship 2.7).

$$\begin{bmatrix} \beta_x = 1 \cdot \cos(\pi/2 + \varphi_{10}) = -\sin\varphi_{10} \\ \beta_y = 1 \cdot \sin(\pi/2 + \varphi_{10}) = \cos\varphi_{10} \\ \beta_z = 1 \cdot \cos(\pi/2) = 1 \cdot 0 = 0 \end{bmatrix} \tag{2.7}$$

The unit vector of the rotated axis O_1z_1 has in rapport of the old system of axes $x_0O_0y_0z_0$ (translated into O_1 without rotation), coordinates data unit vector column (relationship 2.8).

$$\begin{bmatrix} \gamma_x = 1 \cdot \cos 90^0 = 1 \cdot 0 = 0 \\ \gamma_y = 1 \cdot \cos 90^0 = 1 \cdot 0 = 0 \\ \gamma_z = 1 \cdot \cos 0^0 = 1 \cdot 1 = 1 \end{bmatrix} \tag{2.8}$$

ISSN: 1415-7314
ISSN online: 2317-6717

See the obtained matrix T_{01} (relationship 2.2).

Transition from the coordinate system $x_1O_1y_1z_1$ to the coordinate system $x_2O_2y_2z_2$ is done in two distinct phases. The first phase is a translation of the entire system so that (axes being parallel with them itself) the center O_1 to move into the center O_2; then the second stage in which it done the rotation of system of axes, and the center O remains fixed permanently.

The translation of the system from point 1 to the point 2 (see the relationship 2.9) is doing by the column vector, matrix A_{12}.

$$A_{12} = \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} \qquad (2.9)$$

On the old O1x1 axis O2 has been moved with d1, on the old axis O1y1 O2 has been moved with a2, and on the old O1z1 axis O2 has not been moved.

The unit vector of the O_2x_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated but not rotated) the next coordinates (expression 2.10).

$$\alpha_x = 1; \quad \alpha_y = 0; \quad \alpha_z = 0 \qquad (2.10)$$

The unit vector of the O_2y_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated in O_2 but not rotated) the next coordinates (expression 2.11).

$$\beta_x = 0; \quad \beta_y = 0; \quad \beta_z = 1 \qquad (2.11)$$

The unit vector of the O_2z_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated in O_2 but not rotated) the coordinates given by the expression 2.12.

$$\gamma_x = 0; \quad \gamma_y = -1; \quad \gamma_z = 0 \qquad (2.12)$$

The transfer square matrix (the rotation matrix: T12) is writing with relationship 2.13.

$$T_{12} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad (2.13)$$

Transition from the coordinate system $x_2O_2y_2z_2$ to the coordinate system $x_3O_3y_3z_3$ is done in two distinct phases. The first phase is a translation of the entire system so that (axes being parallel with them itself) the center O_2 to move into the center O_3; then the second stage in which it done the rotation of system of axes, and the center O_3 remains fixed permanently.

ISSN: 1415-7314
ISSN online: 2317-6717

First O_2 is moving into O_3 (axes being parallel with them itself; see the relationship 2.14).

$$A_{23} = \begin{bmatrix} d_2 \cdot \cos\varphi_{20} \\ d_2 \cdot \sin\varphi_{20} \\ -a_3 \end{bmatrix} \qquad (2.14)$$

Then O_3 remains fixed, and the axes of coordinate system are rotating. The unit vector of the O_3x_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the α coordinates (see expression 2.15):

$$\alpha_x = 1; \quad \alpha_y = 0; \quad \alpha_z = 0 \qquad (2.15)$$

The unit vector of the O_3y_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the β coordinates (see relationship 2.16):

$$\beta_x = 0; \quad \beta_y = 1; \quad \beta_z = 0 \qquad (2.16)$$

The unit vector of the O_3z_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the γ coordinates (see relationship 2.17):

$$\gamma_x = 0; \quad \gamma_y = 0; \quad \gamma_z = 1 \qquad (2.17)$$

In the model from the figure 1 the system $x_3O_3y_3z_3$ has not been rotated in rapport of the system $x_2O_2y_2z_2$ (from 2 to 3 held just a translation). In this case the rotation matrix is the unit matrix (expression 2.18).

$$T_{23} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.18)$$

The column vector matrix that positions the point M in the coordinate system $x_3O_3y_3z_3$ is written with relation 2.19.

$$X_{3M} = \begin{bmatrix} x_{3M} \\ y_{3M} \\ z_{3M} \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} \qquad (2.19)$$

ISSN: 1415-7314
ISSN online: 2317-6717

Coordinates of the point M in the system (2) $x_2O_2y_2z_2$ are obtained by a transformation matrix which is having the form (2.20):

$$X_{2M} = A_{23} + T_{23} \cdot X_{3M} \qquad (2.20)$$

First, is performed the matrix product (relations 2.21):

$$T_{23} \cdot X_{3M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} = \qquad (2.21)$$
$$= \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix}$$

Then, will be calculated X_{2M} (relationship 2.22).

$$X_{2M} = A_{23} + T_{23} \cdot X_{3M} =$$
$$= \begin{bmatrix} d_2 \cdot \cos\varphi_{20} \\ d_2 \cdot \sin\varphi_{20} \\ -a_3 \end{bmatrix} + \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} = \qquad (2.22)$$
$$= \begin{bmatrix} d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30} \\ d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \\ -a_3 \end{bmatrix}$$

Coordinates of the point M in the system (1) $x_1O_1y_1z_1$ are obtained by the relationships (2.23-2.25).

$$X_{1M} = A_{12} + T_{12} \cdot X_{2M} \qquad (2.23)$$

ISSN: 1415-7314
ISSN online: 2317-6717

$$T_{12}\cdot X_{2M}=$$
$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30}\\ d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\\ -a_3\end{bmatrix}= \quad (2.24)$$
$$=\begin{bmatrix}d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30}\\ a_3\\ d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\end{bmatrix}$$

$$X_{1M}=A_{12}+T_{12}\cdot X_{2M}=$$
$$=\begin{bmatrix}d_1\\ a_2\\ 0\end{bmatrix}+\begin{bmatrix}d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30}\\ a_3\\ d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\end{bmatrix}= \quad (2.25)$$
$$=\begin{bmatrix}d_1+d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30}\\ a_2+a_3\\ d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\end{bmatrix}$$

Coordinates of the point M in the fixed system $x_0O_0y_0z_0$, are written with the relationships (2.26-2.27, 2.27', 2.28).

$$X_{0M}=A_{01}+T_{01}\cdot X_{1M} \qquad (2.26)$$

$$T_{01}\cdot X_{1M}=\begin{bmatrix}\cos\varphi_{10} & -\sin\varphi_{10} & 0\\ \sin\varphi_{10} & \cos\varphi_{10} & 0\\ 0 & 0 & 1\end{bmatrix}\cdot \quad (2.27)$$
$$\cdot\begin{bmatrix}d_1+d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30}\\ a_2+a_3\\ d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\end{bmatrix}$$

$$T_{01}\cdot X_{1M}=$$
$$\begin{bmatrix}(d_1+d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30})\cdot\cos\varphi_{10}-(a_2+a_3)\cdot\sin\varphi_{10}\\ (d_1+d_2\cdot\cos\varphi_{20}+d_3\cdot\cos\varphi_{30})\cdot\sin\varphi_{10}+(a_2+a_3)\cdot\cos\varphi_{10}\\ d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\end{bmatrix} \quad (2.27')$$

ISSN: 1415-7314
ISSN online: 2317-6717

$$X_{0M} = A_{01} + T_{01} \cdot X_{1M} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} + \tag{2.28}$$

$$+\begin{bmatrix} (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \cos\varphi_{10} - (a_2 + a_3) \cdot \sin\varphi_{10} \\ (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \sin\varphi_{10} + (a_2 + a_3) \cdot \cos\varphi_{10} \\ d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix} =$$

$$= \begin{bmatrix} (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \cos\varphi_{10} - (a_2 + a_3) \cdot \sin\varphi_{10} \\ (d_1 + d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30}) \cdot \sin\varphi_{10} + (a_2 + a_3) \cdot \cos\varphi_{10} \\ a_1 + d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix}$$

X_{0M} is arranged in the form (2.29).

$$X_{0M} = \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} = \tag{2.29}$$

$$\begin{bmatrix} d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \cos\varphi_{10} - a_3 \cdot \sin\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \cos\varphi_{10} \\ d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \sin\varphi_{10} + a_3 \cdot \cos\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \sin\varphi_{10} \\ a_1 + d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \end{bmatrix}$$

The same calculations will be presented now by a direct method (having in view the matrix calculations 2.30).

$$\begin{aligned} X_{0M} &= A_{01} + T_{01} \cdot X_{1M} = A_{01} + T_{01} \cdot (A_{12} + T_{12} \cdot X_{2M}) = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot X_{2M} = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot (A_{23} + T_{23} \cdot X_{3M}) = \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \end{aligned} \tag{2.30}$$

It keeps the relationship (2.30’).

$$\begin{aligned} X_{0M} &= A_{01} + T_{01} \cdot A_{12} + \\ &+ T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \end{aligned} \tag{2.30’}$$

Now, one performs the matrix multiplications from expression 2.30’ (relationships 2.31-2.35).

$$T_{01} \cdot A_{12} = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} = \tag{2.31}$$

$$= \begin{bmatrix} d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} \\ d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} \\ 0 \end{bmatrix}$$

ISSN: 1415-7314
ISSN online: 2317-6717

$$T_{01} \cdot T_{12} = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \tag{2.32}$$

$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix}$$

$$T_{01} \cdot T_{12} \cdot A_{23} =$$

$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_2 \cdot \cos\varphi_{20} \\ d_2 \cdot \sin\varphi_{20} \\ -a_3 \end{bmatrix} = \tag{2.33}$$

$$= \begin{bmatrix} d_2 \cdot \cos\varphi_{10} \cdot \cos\varphi_{20} - a_3 \cdot \sin\varphi_{10} \\ d_2 \cdot \sin\varphi_{10} \cdot \cos\varphi_{20} + a_3 \cdot \cos\varphi_{10} \\ d_2 \cdot \sin\varphi_{20} \end{bmatrix}$$

$$T_{01} \cdot T_{12} \cdot T_{23} =$$

$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \tag{2.34}$$

$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix}$$

$$T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} =$$

$$= \begin{bmatrix} \cos\varphi_{10} & 0 & \sin\varphi_{10} \\ \sin\varphi_{10} & 0 & -\cos\varphi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \\ 0 \end{bmatrix} = \tag{2.35}$$

$$= \begin{bmatrix} d_3 \cdot \cos\varphi_{10} \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{10} \cdot \cos\varphi_{30} \\ d_3 \cdot \sin\varphi_{30} \end{bmatrix}$$

ISSN: 1415-7314
ISSN online: 2317-6717

The expression (2.30') takes the form (2.36).

$$X_{0M}=\begin{bmatrix}0\\0\\a_1\end{bmatrix}+\begin{bmatrix}d_1\cdot\cos\varphi_{10}-a_2\cdot\sin\varphi_{10}\\d_1\cdot\sin\varphi_{10}+a_2\cdot\cos\varphi_{10}\\0\end{bmatrix}+$$

$$+\begin{bmatrix}d_2\cdot\cos\varphi_{10}\cdot\cos\varphi_{20}-a_3\cdot\sin\varphi_{10}\\d_2\cdot\sin\varphi_{10}\cdot\cos\varphi_{20}+a_3\cdot\cos\varphi_{10}\\d_2\cdot\sin\varphi_{20}\end{bmatrix}+$$

$$+\begin{bmatrix}d_3\cdot\cos\varphi_{10}\cdot\cos\varphi_{30}\\d_3\cdot\sin\varphi_{10}\cdot\cos\varphi_{30}\\d_3\cdot\sin\varphi_{30}\end{bmatrix}=\begin{bmatrix}x_M\\y_M\\z_M\end{bmatrix}=$$

$$=\begin{bmatrix}d_1\cdot\cos\varphi_{10}-a_2\cdot\sin\varphi_{10}+d_2\cdot\cos\varphi_{20}\cdot\cos\varphi_{10}-a_3\cdot\sin\varphi_{10}+d_3\cdot\cos\varphi_{30}\cdot\cos\varphi_{10}\\d_1\cdot\sin\varphi_{10}+a_2\cdot\cos\varphi_{10}+d_2\cdot\cos\varphi_{20}\cdot\sin\varphi_{10}+a_3\cdot\cos\varphi_{10}+d_3\cdot\cos\varphi_{30}\cdot\sin\varphi_{10}\\a_1+d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\end{bmatrix} \quad (2.36)$$

By the direct kinematics is obtained Cartesian coordinates x_M, y_M, z_M of the point M (the endeffector) in rapport with the three independent angular displacements φ_{10}, φ_{20}, φ_{30}, obtained using actuators (relationships 2.37-2.38).

$$\begin{cases}x_M=f_x(\varphi_{10},\ \varphi_{20},\ \varphi_{30})\\y_M=f_y(\varphi_{10},\ \varphi_{20},\ \varphi_{30})\\z_M=f_z(\varphi_{10},\ \varphi_{20},\ \varphi_{30})\end{cases} \quad (2.37)$$

$$\begin{cases}x_M=d_1\cdot\cos\varphi_{10}-a_2\cdot\sin\varphi_{10}+\\+d_2\cdot\cos\varphi_{20}\cdot\cos\varphi_{10}-a_3\cdot\sin\varphi_{10}+\\+d_3\cdot\cos\varphi_{30}\cdot\cos\varphi_{10}\\ \\y_M=d_1\cdot\sin\varphi_{10}+a_2\cdot\cos\varphi_{10}+\\+d_2\cdot\cos\varphi_{20}\cdot\sin\varphi_{10}+a_3\cdot\cos\varphi_{10}+\\+d_3\cdot\cos\varphi_{30}\cdot\sin\varphi_{10}\\ \\z_M=a_1+d_2\cdot\sin\varphi_{20}+d_3\cdot\sin\varphi_{30}\end{cases} \quad (2.38)$$

Calculations are performed with absolute angular movements (φ_{10}, φ_{20}, φ_{30}), but the actuators movements do not match (all) with the independent angular movements. They are determined as follows (expressions 2.39):

ISSN: 1415-7314
ISSN online: 2317-6717

$$\begin{cases} \varphi_{10} = \varphi_{10} \\ \\ \varphi_{21} = \varphi_{20} \\ \\ \varphi_{32} = \varphi_{30} - \varphi_{20} \end{cases} \qquad (2.39)$$

The first two actuators relative rotations coincide with the independent rotations (used in calculations), but the third actuator relative rotation is obtained as a difference between two absolute rotations (expressions 2.39). The velocities and the accelerations are obtained by the derivatives of the positions expressions (2.38) in rapport of the time.

3. The inverse geometry and inverse kinematics at a MP-3R

The inverse kinematic [2-8] at the serial robots and systems will be exemplified for the 3R kinematic model (see the Fig. 2).

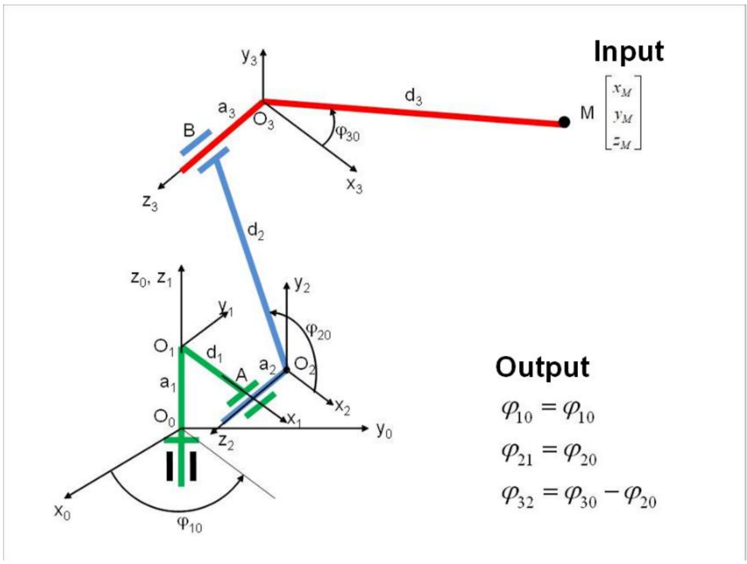

Fig. 2 *The inverse kinematic at the serial robots and systems, exemplified for the 3R model*

In inverse kinematics, one already knows the direct link relationships (3.1), and must determine the inverse relationships, ie to determine the independent rotations φ_{10}, φ_{20}, φ_{30} of the three mobile elements, based on kinematic parameters imposed to the endeffector x_M, y_M, z_M, known (forced).

With the independent determined angles, is then to be calculated the relative rotation movements, of the three driving motors, from the rotating couplers [7].

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$$\begin{cases} x_M = d_3 \cos\varphi_{10} \cdot \cos\varphi_{30} + d_2 \cos\varphi_{10} \cdot \cos\varphi_{20} - \\ - a_3 \sin\varphi_{10} + d_1 \cos\varphi_{10} - a_2 \sin\varphi_{10} \\ \\ y_M = d_3 \sin\varphi_{10} \cdot \cos\varphi_{30} + d_2 \sin\varphi_{10} \cdot \cos\varphi_{20} + \\ + a_3 \cos\varphi_{10} + d_1 \sin\varphi_{10} + a_2 \cos\varphi_{10} \\ \\ z_M = d_3 \sin\varphi_{30} + d_2 \sin\varphi_{20} + a_1 \end{cases} \tag{3.1}$$

Fixed coordinate system was noted with $x_0O_0y_0z_0$. Mobile systems related to the three mobile elements (1, 2, 3) have the indices 1, 2 and 3. Their orientation was chosen conveniently.

System (3.1) is a system of three nonlinear equations (1.1-1.3) with three unknowns (φ_{10}, φ_{20}, φ_{30}) that must be determined; the system 3.1 equations, are rearranging in form that can be seen in the system (3.1 ').

$$\begin{cases} x_M = d_1 \cdot \cos\varphi_{10} - a_2 \cdot \sin\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \cos\varphi_{10} - \\ - a_3 \cdot \sin\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \cos\varphi_{10} (1.1) \\ \\ y_M = d_1 \cdot \sin\varphi_{10} + a_2 \cdot \cos\varphi_{10} + d_2 \cdot \cos\varphi_{20} \cdot \sin\varphi_{10} + \\ + a_3 \cdot \cos\varphi_{10} + d_3 \cdot \cos\varphi_{30} \cdot \sin\varphi_{10} (1.2) \\ \\ z_M = a_1 + d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} (1.3) \end{cases} \tag{3.1'}$$

It aims to solve the system (3.1') directly, to obtain exact and independent solutions.

The first step is multiplying expression (1.1) with ($-\sin\varphi_{10}$) and relation (1.2) with ($\cos\varphi_{10}$); then is summing the two expressions and resulting trigonometric equation (3.2), which can be solved and gives the solutions (3.3-3.4).

$$-x_M \cdot \sin\varphi_{10} + y_M \cdot \cos\varphi_{10} = a_2 + a_3 \tag{3.2}$$

$$\begin{cases} \cos\varphi_{10} = \\ \dfrac{(a_2 + a_3) \cdot y_M \pm x_M \cdot \sqrt{x_M^2 + y_M^2 - (a_2 + a_3)^2}}{x_M^2 + y_M^2} \\ \\ \sin\varphi_{10} = \\ \dfrac{-(a_2 + a_3) \cdot x_M \pm y_M \cdot \sqrt{x_M^2 + y_M^2 - (a_2 + a_3)^2}}{x_M^2 + y_M^2} \end{cases} \tag{3.3}$$

One determines for the first independently parameter (φ_{10}), the trigonometric values of the functions cos and sin ($\cos\varphi_{10}$ and $\sin\varphi_{10}$), (system 3.3).

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We can directly obtain an angle value, when we know sin and cos functions, using expression (3.4).

$$\varphi_{10} = \text{semn}(\sin\varphi_{10}) \cdot \arccos(\cos\varphi_{10}) \qquad (3.4)$$

Angle is given directly by the arccos function.

Sign of sinus (which can be +1 or -1) send the angle in its quadrant, in the top semicircle or the bottom.

The next step is multiplying expression (1.1) with ($\cos\varphi_{10}$) and relation (1.2) with ($\sin\varphi_{10}$); the two resulted are summed, and one obtains the trigonometric equation (3.5).

$$\begin{aligned} & x_M \cdot \cos\varphi_{10} + y_M \cdot \sin\varphi_{10} - d_1 = \\ & = d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30} \end{aligned} \qquad (3.5)$$

This relation (3.5) together with (1.3) form the system (3.6), which generates the independent parameters (φ_{20} *and* φ_{30}, the last).

$$\begin{cases} x_M \cdot \cos\varphi_{10} + y_M \cdot \sin\varphi_{10} - d_1 = \\ = d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30} \quad (3.5) \\ \\ z_M - a_1 = d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \quad (1.3) \end{cases} \qquad (3.6)$$

With notations (3.7) one obtains for the equations system (3.6) the direct and exact solutions (3.8).

The equations (3.6) take the form (3.6’).

$$\begin{cases} C_1 = d_2 \cdot \cos\varphi_{20} + d_3 \cdot \cos\varphi_{30} \quad (3.5') \\ C_2 = d_2 \cdot \sin\varphi_{20} + d_3 \cdot \sin\varphi_{30} \quad (1.3') \end{cases} \qquad (3.6')$$

System (3.6’) can be written in the form (3.6’’).

$$\begin{cases} C_1 - d_2 \cdot \cos\varphi_{20} = d_3 \cdot \cos\varphi_{30} \quad (3.5'') \\ C_2 - d_2 \cdot \sin\varphi_{20} = d_3 \cdot \sin\varphi_{30} \quad (1.3'') \end{cases} \qquad (3.6'')$$

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Equations (3.6’’), squared and added together, give the expression (3.6’’’).

$$K - 2 \cdot C_1 \cdot d_2 \cdot \cos\varphi_{20} = 2 \cdot C_2 \cdot d_2 \cdot \sin\varphi_{20} \qquad (3.6''')$$

$$\begin{cases} C_1 = x_M \cdot \cos\varphi_{10} + y_M \cdot \sin\varphi_{10} - d_1 \\ C_2 = z_M - a_1 \\ k = C_1^2 + C_2^2 + d_2^2 - d_3^2 \end{cases} \qquad (3.7)$$

From equation (3.6’’’) one obtains $\cos\varphi_{20}$, $\sin\varphi_{20}$, and φ_{20} (first relations of system 3.8), and using expressions (3.6’’) it determines then $\cos\varphi_{30}$, $\sin\varphi_{30}$, and φ_{30} (last relations of system 3.8).

$$\begin{cases} \cos\varphi_{20} = \dfrac{k \cdot C_1 \pm C_2 \cdot \sqrt{4 \cdot C_1^2 \cdot d_2^2 + 4 \cdot C_2^2 \cdot d_2^2 - k^2}}{2 \cdot (C_1^2 + C_2^2) \cdot d_2} \\ \sin\varphi_{20} = \dfrac{k \cdot C_2 \mp C_1 \cdot \sqrt{4 \cdot C_1^2 \cdot d_2^2 + 4 \cdot C_2^2 \cdot d_2^2 - k^2}}{2 \cdot (C_1^2 + C_2^2) \cdot d_2} \\ \varphi_{20} = semn(\sin\varphi_{20}) \cdot \arccos(\cos\varphi_{20}) \\ \cos\varphi_{30} = \dfrac{C_1 - d_2 \cdot \cos\varphi_{20}}{d_3} \\ \sin\varphi_{30} = \dfrac{C_2 - d_2 \cdot \sin\varphi_{20}}{d_3} \\ \varphi_{30} = semn(\sin\varphi_{30}) \cdot \arccos(\cos\varphi_{30}) \end{cases} \qquad (3.8)$$

4. Conclusions

Kinematics of the serial manipulators and robots can be illustrated by a 3R kinematic model, a medium difficulty system, ideal for understanding the phenomenon, but also to specify the basic knowledge necessary for starting calculations for systems simpler and more complex.

The paper presents an original geometrical and kinematic method for the study of geometry and determining positions of a MP-3R structure. It presents shortly the MP-3R direct and inverse kinematics, the inverse kinematics being solved by an original exactly method. One presents shortly an original method to solve the robot inverse kinematics exemplified at the 3R-Robots (MP-3R).

If one study (analyze) an anthropomorphic robot with three axes of rotation (which represents the main movements, absolutely necessary), we already have a base system, on which one can then add other movements (secondary, additional). Calculations were arranged and in the matrix form.

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5. Discussion

Kinematics of the anthropomorphic systems may be solved by a basic model 3R spatial by matrix calculations (which were presented on this work), or on a 2R planar, simplified model [9].

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