A MULTICRITERIA DECISION MODEL FOR ELECTORAL PREFERENCES

UM MODELO DE DECISÃO MULTICRITÉRIO PARA PREFERÊNCIAS ELEITORAIS

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ABSTRACT

In this article, a multicriteria model for the assessment of electoral preferences is built. The multicriteria methodology used is probabilistic composition of preferences, extended to consider possible interactions. The main objective of the article is to demonstrate the viability, the social interest, as well as the attractiveness for the political parties, of the new approach to electoral preferences assessment. The main hypothesis underlying the model is that the population as a whole evaluates each party by considering its position with respect to a set of issues. An analysis is developed of the case of parties’ positions and voters’ preferences set on the basis of government spending distribution. A principle of preference concentration is applied to explain criteria importance. The influence of group interests, though not assumed to be determinant, is not denied. An expected use of the model is for parties to evaluate alliance strategies, since the probabilistic modeling of the preferences and of the direct treatment of the possible interactions among criteria opens a new perspective for the analysis of electoral allegiances. A theoretical example is analyzed. An application involving real preferences in a Brazilian election is discussed. This example and this application demonstrate the effectiveness of the approach.

Keywords: Coalitions; Composition of Probabilistic Preferences; Government Spending; Multicriteria Decision Analysis; Voting Model
RESUMO

Neste artigo, é construído um modelo multicritério para a avaliação de preferências eleitorais. A metodologia multicritério empregada é a composição probabilística de preferências, estendida de modo a considerar possíveis interações. O objetivo principal do trabalho é demonstrar a viabilidade, interesse social e atratividade para os partidos políticos da nova abordagem para a avaliação de preferências eleitorais. A principal hipótese subjacente ao modelo é que a população como um todo avalia cada partido considerando sua posição em relação a um conjunto de temas. É desenvolvida uma análise do caso em que as posições dos partidos e as preferências dos eleitores são definidas com base na distribuição dos gastos do governo. Um princípio de concentração de preferências é aplicado para explicar a importância dos critérios. A influência dos interesses de grupo é admitida, embora não assumida como determinante. Uma utilização esperada do modelo é na avaliação de estratégias de aliança pelos artigos. A modelagem probabilística das preferências e o tratamento direto das possíveis interações entre critérios abre uma nova perspectiva para a análise de coalizões eleitorais. Um exemplo teórico é analisado. Uma aplicação envolvendo preferências reais em uma eleição brasileira é discutida. Este exemplo e esta aplicação demonstram a efetividade da abordagem.

**Palavras-chave:** Apoio à Decisão Multicritério; Coalizões; Composição Probabilística de Preferências; Gastos do Governo; Modelo de Votação
1 INTRODUCTION

This article examines competition and alliances among political parties from a new perspective based on the employment of multicriteria decision analysis to model voter preferences in an integrated way. Multicriteria decision analysis can insert into negotiation models a community’s viewpoint, in addition to the viewpoints of the conflicting parties.

The basic assumption of the model is that the parties seek voters’ approval and that the voters’ choice is based on their integrated opinion about a certain number of issues on which parties and voters take proper positions. Each party seeks to have the electorate agree with its position on each issue, and voters rate parties on the basis of their likelihood of agreeing with them on these issues.

Electoral decisions can be thought of as the outcome of a conflict in which preferences for the parties are determined by how well their propositions satisfy group interests (Schwartz-Shea & Simmons, 1991). However, modeling the election as a decision in which each voter evaluates the parties according to their positions with respect to a set of interacting issues of general interest allows for ampler analysis. The relevance of particular interests needs not be denied, since group interests may be included among the interacting issues.

Thus, in this model, the number of votes cast for each party does not result from matching the preferences of groups of supporters. It results from the application of a composition rule substantiated by a collective perspective.
The results from the application of these composition rules can be seen as constituting predictions of the result of a future election. By comparing the results predicted from the application of different composition rules, parties can revise their positions and forge alliances that modify the space of alternatives offered to voters. This may be decisive in preventing groups uncommitted to democracy from taking power by winning elections with many candidates and insufficient voter enlightenment.

Indeed, in representative democracy, when election preferences are built around conflicts between societal segments and sectarianism prevails over awareness, the possibility of politicians asserting dictatorial powers by claiming to be acting in the interests of a certain group grows. On the other hand, if elections are approached in an environment where objective assessments of each party's likelihood of winning are available and an early review of the political framework is possible, democracy is less vulnerable.

More often, the schism in majoritarian elections creates an opportunity for minority currents to seize control of the Executive chief officer position. This causes management difficulties that can be avoided if forecasting the electoral prospects of parties with similar positions on the modeled criteria results in alliances that more accurately reflect the will of the voters.

The perspective adopted here does not consider the voters separated into antagonistic factions represented by political parties. Instead, the parties are alternatives represented by their stances on relevant themes, and people determine their preferences holistically. So we arrive at a generic voter that
combines evaluations via a Choquet capacity (Choquet, 1953). This capacity is determined by assigning priorities not only to the individual criteria, but also to the sets of criteria, taking into account the interaction between them.

The criteria receive priorities defined according to their ability to indicate the best alternative. The inclusion in the model of the interaction between the criteria allows for consideration of a large variety of social values. Additionally, it enables to incorporate shifts to new political relationships into the analysis as they occur. This is particularly interesting to highlight the multiple features of sustainability management.

In a broader perspective, a hypothesis of this study is that framing the electoral problem in terms of public policy stances rather than party preferences lifts the collectivity values above the interests of party competition. It is expected that by developing this form of assessing electoral preferences voters will gain a more nuanced understanding of their party preferences. Thus, the main goal of the work is to establish, not only the viability of a novel approach to electoral preferences modeling, but also its social utility, and its appeal for the political parties.

To develop this approach more concretely, the confrontation between political parties is examined by considering their positions regarding public spending. Four positions are identified: liberal, conservative, progressive and socialist.
Composition of probabilistic preferences (CPP) (Sant’Anna, 2015) is the methodology employed to derive, from the evaluations of the parties according to their positions with respect to the multiple interacting issues, the global preference for each party. The solutions obtained through the application of CPP composition rules based on probabilities of preference can be viewed as approximations to what would constitute requests in the best interests of the collectivity.

2 LITERATURE REVIEW
The notion that voters choose the party that minimizes the discrepancy between their assessment of a set of pertinent issues and the party’s stances on those issues has been extensively explored in the literature (Downs, 1957; Sengupta and Sengupta 2008; Brusco, Dziubinski and Roy 2012). Downs (1957) developed an approach to modeling voting based on the position of voters and parties along a line of expectations about government size. Parties attract followers based on a position along the line from smaller to larger government. This approach underpins a fruitful line of thought, mainly because it suitably describes the case of two parties and segmentation of voters with strong fidelity to each party. This segmentation allows for modeling the behavior of the population on the basis of uniform individual behavior within each segment. Voters are divided according to what their parties profess people should expect from the government.

Downs’ model predicts electoral results by starting from this segmentation of voters by party. In line with the microeconomic development of Hotelling (1929), each voter votes for the candidate or party...
that is closest to his or her political position. Thus, in the dispute between two parties, when one party takes a position to the right of the other, it obtains the votes of all voters on the right side. This drives the parties to become ideologically closer to each other because, even starting from distant principles, they modify their programmatic propositions to satisfy a number of social segments that encompass the largest possible number of votes, what causes the distinction between them to become increasingly blurred.

This would lead each segment’s gain derived from its party's electoral victory becoming insufficient to justify voting costs. A paradox is then signaled when, in countries with bipartisanship, voter turnout does not decrease. This has sparked speculation about the role of altruistic motivations in electoral behavior (Fowler, 2006; Edlin et al, 2008; Chiang, 20212).

No approach, however, models the relationship between the preference for parties and the ability of these parties to contemplate their country interests, nor to aggregate the derived preferences. The paradox extends to models that incorporate a large number of directions connecting the various elements of the universe of parties (Huberty 2013; Jungherr et al., 2017).

Empirical research has revealed the difficulty in fitting electoral results with this model when the number of parties is larger or fidelity to parties is weaker. To address these difficulties, a large spectrum of variants of the model has been developed. These developments have been especially useful
for extending Down’s approach to situations with more than two competing parties culled by proper primary election protocols (Adams, Merrill and Grofman, 2005; Shefield and Sened, 2006).

Different psychological developments treated voting from a consumption perspective (Crain, 1977; Chapman and Palda, 1981) or an identity perspective (Campbell et al., 1960; Fiorina, 1981). The adoption of types of spending as the factors determining voting decisions has also been the subject of research, as has the association of government spending with ideological positions (Jost, 2017; Sousa, Paulo and Marôco 2017, for instance). Other analyses of the motivation to vote have been added, addressing different economic motives or social pressures to vote (Jenke and Huettel, 2017; Dacrema and Benati, 2020).

All such approaches keep the premise that preferences for parties in elections result from the composition of the preferences of social segments associated with different levels of commitment to the various parties.

The ineffectiveness of the view based on an electorate fractionation is frequently explained by the increasing importance of a variety of broad issues, involving, for instance, anti-establishment propositions and environmental protection (McGregor et al., 2015; Abramowitz and Webster, 2016; Mayer, 2017; Kurze and Lenschow, 2018; Abramowitz and McKoy, 2019). Another factor is the need to take into account interaction between development goals to keep sustainable the economic systems (Robinson and Robinson, 2017; Massa et al, 2018; Vorrabler and Müller, 2019). As a consequence,
party preferences would become less formal and change rapidly (Mainwaring and Scully, 1995; Sprenkels, 2019).

While the cultural segmentation view is prevalent, the importance for society of factors not contemplated in traditional party platforms brings detrimental practical consequences. It serves to legitimize the dilution of preferences among a large number of parties, some of which take ambiguous positions that sometimes resemble what was dubbed “The Marsh” in the French Revolution. Politicians of The Marsh, who have survived over time through personal clientelist behavior (Mainwaring, 1999; Kitschekt and Wilkinson, 2007), always join the winning side following an election and never appear capable of winning a general election on their own. In elections where there is insufficient time for the formation of mature opinions, such politicians can garner popular support by professing to represent groups with specific interests related to current events. After an electoral victory, they implement antidemocratic policies and establish dictatorships, wreaking havoc on the country's social and economic development (Cohen and Smith, 2016; Waisbord, 2018).

3 METHODOLOGY
To consider the effects on the voters’ preferences of the presence of parties with varying positioning on a set of issues of inconstant and interacting effects, the approach here developed starts from preferences for each party according to multiple criteria and allows for the entrance and withdrawal of parties.
To provide an example, a model taking government size as the basic issue underlying opinion formation is developed. In this model, even though Downs’s model foundation still applies, what decides electoral outcomes is the prevalence of global preferences on the subject of government actions.

Voters’ and parties’ views intersect along a line measuring the extent of government action, but there is no necessary political division of the population into determinant fractions. The country's population shifts its preferences homogeneously as an integrated body with a unified perspective. The parties seek to exploit interests in terms of economic class, geographic region or religion, for instance, but their power to attract votes is built through a process of validating values of the electorate as a whole.

3.1. Composition of Probabilistic Preferences

Composition of probabilistic preferences (CPP) is the multicriteria method applied to combine the electoral preferences. CPP is based on the recognition of imprecision in the preference elicitation process due to the possibility of measurement errors and to the varying value that the same measure of any attribute may have in different situations.

CPP works by using numerical values of evaluations of alternatives according to multiple criteria as the basis for calculating vectors of probabilities of each alternative being the most preferred according to each criterion. Joint probability calculation rules are then used to derive final scores from these vectors of preference probabilities.
Probabilistic assessments help decision-makers understand the variability in their own preferences and their proximity to others’ preferences. A better understanding of preferences may lead to reformulate the set of alternatives, agents uniting around the more viable options. This reduces the number of conflicting alternatives and increases the possibility of reaching a consensual solution (Sant’Anna, Costa and Silva, 2021).

The combination of the probabilistic preferences along the multiple criteria can be achieved by weighted average, with the weights of the criteria being derived from the results of the evaluation of the alternatives according to each of them. This derivation involves an application of the principle of preference concentration (Sant’Anna and Sant’Anna, 2019). According to this principle, the decision-maker seeks to maximize the probability of identifying the most preferred alternative. The basis for this application of the principle of concentration of preferences is in the idea that the decision-maker tends to give greater importance to those criteria most capable of identifying an alternative as the one with the highest preference.

The importance of any criterion is then derived from its ability to identify an alternative as the most preferred. Thus, analogously to, when applying an isolated criterion, the preference for an alternative is given by its probability of being the most preferred among the available alternatives, so the preferences for the criteria are assessed by comparing the maximum probabilities of preference that they attribute to the possible alternatives.
3.2 Interaction

In a classic multicriteria analysis, to forecast the outcome of a forthcoming election, the electorate would be divided into separate segments. The weight of each criterion would be determined by the fraction of the electorate who wish to use it. The probability of the voters in each segment voting for a particular party would be given by the probability that that party would be the best option according to the criterion associated with the segment in question.

The forecast would then disregard any possible interaction between the criteria and would be given by the weighted average of these probabilities, with weights determined by estimates of the proportions of the population in each segment.

In the approach here explored, rather than being split, the electorate forms its preferences cohesively. This leads to the calculation of the score for each party by computing the Choquet integral of its preference probabilities with respect to a Choquet capacity. The electorate as a whole votes for the party with the most attractive set of evaluations, with recognition of the inaccuracy in the evaluation by each voter of the actual positions and of the effects of interactions.

A necessary condition for using the Choquet integral is the commensurability of the criteria. This is guaranteed by the transformation of the original assessments into preference probabilities.
To obtain the Choquet capacity, priorities may be gathered directly from decision-makers. This is even more difficult to accomplish than obtaining weights for the isolated criteria because it requires determining the relative importance of each set of criteria.

Instead, the principle of concentration of preferences is used once more here to indirectly determine the capacity. It leads to measure the interaction between any two criteria by the increase in the maximum probability of preference when the pair of criteria is considered instead of just one criterion alone. This rule is extended to pairs of sets containing any number of criteria; the comparison is then made between the maximum preference for each set separately and for the two sets together.

The score by the Choquet integral of an alternative with divergent assessments by criteria with positive interaction is decreased, whereas it is increased if its evaluations are convergent according to such criteria. This means that when the principle of concentration of preferences is used to build the capacity, the score is increased for those alternatives jointly most preferred by different criteria.

The real effect of the interaction can be assessed by means of the Shapley values of the criteria (Shapley, 1953). The sum of the Shapley values is one, and the Shapley value of each criterion increases with its positive interaction with the other criteria.

Shapley values can be applied to articulate the application of the principle of concentration of preferences with other forms of assigning importance to the criteria. If, after calculating the capacity, the Shapley values of the criteria are found to contradict some probability distribution in the set of
criteria postulated by a desirable assignment rule, overvaluing one set of criteria and undervaluing others, rectification factors can be applied.

The calculations made in the following analysis employ the CPP (Gavião et al., 2018) and kappalab (Grabisch and Kojadinocic, 2005) packages in R (R Core Team 2019).

3.3 The government size decision framework
The objective of the multicriteria modeling proposed here is to provide an objective assessment of the probabilities of winning for each party, which opens the possibility of revising the political framework before elections occur.

The parties are the alternatives. They are represented by their evaluations according to criteria related to their position on themes that attract voters’ attention.

Here it is assumed that the issues that dominate the political debate basically refer to the size of the government. Deeper motivations, such as, on the one hand, fear of facilitating the embezzlement of public resources or, on the other, the hope that government spending will accelerate economic and social development, lead citizens to place themselves politically around positions regarding the size of different components of the public budget.

The conflicting positions refer to the willingness to increase or reduce three types of spending: investment in economic infrastructure, other kinds of investment with lower economic impact, and
consumption and transfer payments. The criteria evaluate the position of the parties with respect to each kind of spending.

It is initially assumed that there are four parties with positions determined by their spending proposals at each of the three levels: the liberal, conservative, progressive and socialist parties. The first, the liberal party, is defined by the desire to reduce the government by minimizing all types of spending. The second, the conservative party, is defined by proposing the minimization of spending on investments in other sectors of the economy and consumption expenditures and transfer payments, with a maximization of investment in economic infrastructure. The third, the progressive party, wants to minimize the cost of consumption and transfers but would be willing to maximize all types of spending for investments, both in economic infrastructure and other sectors. Finally, the fourth party is characterized by a desire to maximize government presence through all three types of spending.

To represent these positions by numerical values of multiple attributes in a general assessment, each party is represented by a set of possible volumes for each type of spending, measured on a scale from 0, representing the minimum possible spending, to 1, the maximum. When a party opposes a given type of spending, its spending volume is associated with a triangular distribution in the interval [0, 1] with 0.1 mode. On the other hand, when the party is favorable to a given type of spending, its mode is 0.8. The reason for this asymmetry (the 0.1 mode is closer to the lower end than the 0.8 mode to the upper end) is the greater ease of growth in the maximum, due to the greater possibility of creating new
expenses. At the other extreme, spending reductions are bounded by more concrete limits. This larger distance of 0.8 to the upper limit than 0.1 to the lower bound plays an important role in the probabilistic analysis.

Assuming that one electoral issue prevails, support for each party is proportional to the likelihood that the party's position on that issue coincides with the voters’ preference. Thus, assuming, for instance, a concentration in maximizing spending on investments in economic infrastructure, the preference for each party is given by the probability that the volume of such spending in that party’s proposals is the highest among the four parties.

4 RESULTS
The proposed approach is here applied within the context of public spending evaluations. The electorate would be divided into six segments, defined by their desire to maximize or minimize each of the three types of spending. Thus, six issues—maximizing or minimizing each of three kinds of spending—are accepted as the relevant criteria for modeling the context in which the electorate's decision is developed: Crit. 1: maximization of economic infrastructure spending, Crit. 2: maximization of other investment spending, Crit. 3: maximization of consumption and transfer payment spending, Crit. 4: minimization of economic infrastructure spending, Crit. 5: minimization of other investment spending, and Crit. 6: minimization of consumption and transfer payment spending.
4.1 Withdrawal Example

Table 1 presents data for a numerical analysis. The alternatives are identified by the modal values of the spending distributions proposed by each party for each type of spending.

Table 1. Identification of the Four Alternatives

<table>
<thead>
<tr>
<th>Party</th>
<th>Economic Infrastructure</th>
<th>Other Investment</th>
<th>Consumption and Transfers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Conservative</td>
<td>0.8</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Progressive</td>
<td>0.8</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Socialist</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 2 shows the estimated probability that each alternative maximizes or minimizes the amount of each type of spending, rounded to the third decimal place.

Table 2. Probabilities of Maximizing and Minimizing Spending

<table>
<thead>
<tr>
<th>Party</th>
<th>Crit. 1</th>
<th>Crit. 2</th>
<th>Crit. 3</th>
<th>Crit. 4</th>
<th>Crit. 5</th>
<th>Crit. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal</td>
<td>0.072</td>
<td>0.098</td>
<td>0.147</td>
<td>0.576</td>
<td>0.411</td>
<td>0.313</td>
</tr>
<tr>
<td>Conservative</td>
<td>0.309</td>
<td>0.098</td>
<td>0.147</td>
<td>0.141</td>
<td>0.411</td>
<td>0.313</td>
</tr>
<tr>
<td>Progressive</td>
<td>0.309</td>
<td>0.401</td>
<td>0.147</td>
<td>0.141</td>
<td>0.089</td>
<td>0.313</td>
</tr>
<tr>
<td>Socialist</td>
<td>0.309</td>
<td>0.401</td>
<td>0.558</td>
<td>0.141</td>
<td>0.089</td>
<td>0.061</td>
</tr>
</tbody>
</table>

The application of the composition rule to the preference probabilities for each party leads to the scores shown in Table 3. It reveals that the preferences of the electorate are greater for the liberal party, with a score of 0.494, followed closely by the socialist party, at the other extreme, with a score of 0.472. Political positions with spending preferences varying along the types of spending receive scores closer to each other and away from the scores of the other two.
Table 3. Scores of the Four Parties

<table>
<thead>
<tr>
<th>Party</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal</td>
<td>0.494</td>
</tr>
<tr>
<td>Conservative</td>
<td>0.328</td>
</tr>
<tr>
<td>Progressive</td>
<td>0.321</td>
</tr>
<tr>
<td>Socialist</td>
<td>0.472</td>
</tr>
</tbody>
</table>

The Shapley values of the six criteria are presented in Table 4. Table 4 shows how the principle of preference concentration places greater importance on the criteria with the highest extreme values in Table 2, which are 0.576, for Crit. 4, and 0.558 for Crit. 3. They receive Shapley values of 0.228 and 0.212, respectively.

Table 4. Shapley Values of the Model of Four Alternatives

<table>
<thead>
<tr>
<th>Crit. 1</th>
<th>Crit. 2</th>
<th>Crit. 3</th>
<th>Crit. 4</th>
<th>Crit. 5</th>
<th>Crit. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.124</td>
<td>0.149</td>
<td>0.212</td>
<td>0.228</td>
<td>0.159</td>
<td>0.128</td>
</tr>
</tbody>
</table>

The proximity between the final preference scores for the liberal party and the socialist party suggests that reducing the number of parties might change the choice. For example, if the progressive party prefers the socialist extreme to the liberal extreme, it can simply withdraw from the dispute. In this case, the preference probabilities become those in Table 5, leading to the final scores in Table 6. This table demonstrates that an alliance between those who want reduced costs but high levels of investment and those who want a large government defeats those who want a smaller government.

Table 5. Probabilities of Maximization and Minimization with Three Parties

<table>
<thead>
<tr>
<th>Party</th>
<th>Crit.1</th>
<th>Crit.2</th>
<th>Crit.3</th>
<th>Crit.4</th>
<th>Crit.5</th>
<th>Crit.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal</td>
<td>0.112</td>
<td>0.178</td>
<td>0.178</td>
<td>0.653</td>
<td>0.449</td>
<td>0.449</td>
</tr>
<tr>
<td>Conservative</td>
<td>0.444</td>
<td>0.178</td>
<td>0.178</td>
<td>0.173</td>
<td>0.449</td>
<td>0.449</td>
</tr>
<tr>
<td>Socialist</td>
<td>0.444</td>
<td>0.644</td>
<td>0.644</td>
<td>0.173</td>
<td>0.102</td>
<td>0.102</td>
</tr>
</tbody>
</table>
Table 6. Scores of Three Parties

<table>
<thead>
<tr>
<th>Party</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal</td>
<td>0.571</td>
</tr>
<tr>
<td>Conservative</td>
<td>0.413</td>
</tr>
<tr>
<td>Socialist</td>
<td>0.619</td>
</tr>
</tbody>
</table>

The Shapley values after the withdrawal, in Table 7, show the reduction in the importance of Crit. 4. The Shapley values for Crit. 2 and Crit. 3, whose values remain the same, become the highest.

Table 7. Shapley Values with Three Alternatives

<table>
<thead>
<tr>
<th>Crit. 1</th>
<th>Crit. 2</th>
<th>Crit. 3</th>
<th>Crit. 4</th>
<th>Crit. 5</th>
<th>Crit. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.143</td>
<td>0.201</td>
<td>0.201</td>
<td>0.193</td>
<td>0.131</td>
<td>0.131</td>
</tr>
</tbody>
</table>

If, finally, we continue to reduce the number of alternatives, also removing the conservative party and leaving only the liberal and socialist extremes, the symmetry of the construction results in identical capacities for all unitary sets, as well as for the Shapley values, and, thus, identical scores for the two alternatives.

The Shapley values in Tables 4 and 7 call attention to another possible development of the analysis. The low Shapley values of the criteria measured by the probabilities of maximizing spending on economic infrastructure and of minimizing consumption and transfer expenses go against the progressive and the conservative perspectives. Nevertheless, these alternatives may be more relevant to the analyst than the other criteria in the analysis. This suggests resorting to the artifice of rectifying the
input values in the capacity determination. Vector of rectification factors with a reduction for the other four criteria were applied, after or before the removal of the progressive alternative, and the change in the scores obtained did not affect the ranks.

4.2 Application to a local election

An application involving real preferences is described here. It starts with the development of a method for initial assessment of data for application of a probabilistic model.

This method is used to derive positions relative to spending in the first round of the 2020 mayoralty election in Rio de Janeiro, Brazil’s second largest city. In 2020 there were neither statewide nor federal elections in Brazil (municipal elections are staggered every four years with national/state elections). More than 3 million people voted in this election.

The positions were assessed by counting the number of promised actions involving significant positive expenses and spending cuts proposed in each party’s platform. These total numbers were weighted according to the party’s free radio and TV propaganda time.

First, the difference between the numbers of positive and negative initiatives of each type of spending in the candidate’s program was counted. Then, for each type of spending, a median of the set of counts of the parties was determined and subtracted from each spending balance. This leads to a positive or negative value that indicates the position of the party with respect to that type of spending.
Finally, this difference is adjusted through multiplication by the number of minutes of free radio and TV electoral propaganda time granted to the party. These corrected differences constitute the entrance matrix for computing the probabilities of maximizing and minimizing the preference for each type of spending.

This last correction tries to counteract the lower importance assigned to the written program by the strongest parties. According to the Brazilian law, the parties receive campaign funds and free radio and TV propaganda time according to their present number of representatives in the lower house of the National Congress. Only the parties that elected representatives in at least 9 of the country’s 27 states (with the Federal District counted as a state) in the last national election are contemplated in this time division. Any other media exposure must respect equalitarian rules.

Thus, the size of the federal representation and the radio and TV time that it determines constitute a main determinant of access to means of exposing the parties’ proposals and of making their principles known by the population. The parties’ written platforms constitute only a conceptual source of such exposition.

Though the platforms are exhibited in the website of the Electoral Justice System, parties with lower radio and TV time usually pay stronger attention than the others to making their position visible in the written platforms. The stronger parties avoid formally submitting detailed goals. For instance, the Democratic Movement Party, the most traditional party, with the highest number of municipal mayors
in the country, did not submit a written platform in Rio de Janeiro’s contest. It thus entered the procedure for the assessment of the spending preferences with the central value.

The adjusted balances of initiatives on each of the three spending axes represent the candidates in a three-dimensional positional vector. These vectors gave rise to the columns of the entrance matrix for the probabilistic composition. This matrix of probabilities of maximization and minimization produced the Choquet capacity and finally the Choquet integrals.

Table 8 shows the parties’ positions asserted by the signs on their spending proposal balances, daily minutes of free radio and TV exposure, spending proposals’ isolated scores, final composed scores and proportion of valid votes obtained in the first round of the election.

<table>
<thead>
<tr>
<th>Party</th>
<th>Position</th>
<th>Daily min.</th>
<th>Econ.</th>
<th>Infr.</th>
<th>Other Inv.</th>
<th>Other Spend.</th>
<th>Final Score</th>
<th>Votes %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Democrats</td>
<td>progressive</td>
<td>125</td>
<td>3</td>
<td>8</td>
<td>-2</td>
<td>0.37</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>Republicans</td>
<td>conservative</td>
<td>122</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>0.31</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>Democratic Labor</td>
<td>socialist</td>
<td>71</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>0.09</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Workers</td>
<td>socialist</td>
<td>75</td>
<td>3</td>
<td>3</td>
<td>14</td>
<td>0.27</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Liberal Social</td>
<td>Liberal</td>
<td>100</td>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>0.15</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Socialism&amp;Liberty</td>
<td>socialist</td>
<td>16</td>
<td>6</td>
<td>6</td>
<td>17</td>
<td>0.08</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Democratic Mov.</td>
<td>conservative</td>
<td>43</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Christian Social</td>
<td>Liberal</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0.10</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Republican Social</td>
<td>Liberal</td>
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<td>1</td>
<td>2</td>
<td>0</td>
<td>0.09</td>
<td>0.01</td>
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</tr>
<tr>
<td>Republican Social</td>
<td>socialist</td>
<td>14</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>0.08</td>
<td>0.00</td>
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</table>
The ordering of preferences obtained indicated that, even in the presence of the economic and social problems caused by the COVID-19 pandemic that plagued the city during the campaign period, the composition of the government budget had a significant role in the parties’ positions and the preferences of the population. The Spearman rank correlation coefficient of the model’s final scores and the percentages of votes of the candidates in the election in the two last columns of Table 8 are greater than 95%.

Presently, a feature of Brazilian politics is the competition among parties in the same ideological range. Besides the four parties identified in the second column of Table 8 as socialist, four more socialist parties, without enough representation in the Congress to allow for free radio and TV presence, presented candidates in the Rio de Janeiro 2020 election. Due to their limited access to public opinion, they were not considered in the analysis.

In Brazilian elections for president, governor, or mayor of a large city, if the candidate obtaining the largest number of votes does not receive more than 50%, the two top vote-receivers pass to a second round (with equal exposure time). Prior to the start of the election campaign, efforts were made to withdraw candidacies in the socialist field. These efforts were frustrated by the lack of sufficiently objective evaluations of preference, as different analyzes of the conjuncture indicated greater chances of success for different candidates. The results in Table 8 highlight the possibility that an alliance between the socialist parties might enable one of them to reach the second round.
5 DISCUSSION
The two examples provided above illustrated the viability of the proposed modeling approach. In the first example, a broad formulation was devised that could be applied to the realities of large and small countries, developed and developing regions, and even national or international institutions. The second permitted the introduction of local management features.

The probabilistic assessment allowed for the incorporation of two essential components of electoral preferences formation in reality that are absent in direct counting approaches. The first is the existence of preferences for particular parties as a result of their radically opposite positions on a same issue. The second is the occurrence of interactions between positions on distinct issues, as well as between opposite positions on the same issue. Both of these were clearly strained in the examples.

The model's core premise is that voters’ beliefs are, in nature, homogeneous, not segmented. The two examples highlighted how, rather than focusing on the political parties’ rivalry, the electoral contests may prioritize the assignment of significance to topics of public concern.

Rather than using a weighted average of the particular interest assessments, the application of CPP follows a general interest principle that assigns importance to the criteria according with the purpose of preference concentration. This element of the paradigm played a key role in motivating the parties to form coalitions.
Thus, the results obtained provided significant evidence that democracies, at various stages of economic, social and political development, may benefit from the elevation of the electoral debate enabled by the proposed modeling approach.

6. **FINAL COMMENTS**

This article proposes an innovative approach to modeling electoral preferences. This modeling approach extracts from the initial assessments of preferences for the parties values for the importance and interaction among criteria, which may be adjusted, as well as the set of candidacies being compared. This provides an objective framework for the analysis of coalitions.

The modeling of preferences related to each issue is based on the standard hypotheses of unimodality and linear decay of the preferences distribution. The assignment of importance to issues also follows a straightforward principle of preference concentration maximization. These simplifications proved capable of producing satisfactory results and of guiding the formation of parties' strategic partnerships. If more information is available, more complex hypotheses can be applied.

Analyses driven by the composition of the government budget are developed. Preferences are presumed to be determined by positions concerning public spending. Three distinct categories of expenditures are considered. In future developments, a greater number may be considered. There is no need to order them. Furthermore, monetary quantifications of the positions are not required. Hence, the criteria may include specific preferences for other government actions.
Rather than focusing exclusively on fiscal concerns, the model might incorporate a broad range of issues affecting vote preferences from a social, political, environmental, or cultural perspective. More comprehensive models might consider, in addition to the classic left-right opposition over whether to increase or decrease government social spending and about the presence of economic class representation in politics, criteria relating to questions like the global climate change and the preservation of ecological diversity for the future generations, as well as to the debate over the priority of maintaining public order over human rights and civilizability. The decision to restrict the scope to the area of economic management was motivated by the intention of facilitating the examination of the influence of the criteria's interactions. The relationships between the other subjects mentioned above are simpler.

More comprehensive analyses can be developed not only by including additional criteria, but also by more thoroughly modeling the alternatives, either by including more parties or by quantifying the evaluations through the use of surveys of preferences effectively observed in time and space-defined situations. Classifications based on these localized opinion polls can be utilized to establish both the positions on each issue and the importance accorded to the issues.

The integration of all these factors in the model will surely create conditions for a more reliable analysis of political perspectives. If models of this type become ingrained in political culture, they will
enhance the emergence of new dynamics in electoral compositions and the capacity of voters to make better informed electoral choices.

REFERENCES


